Microfoundations of DSGE Models: I Lecture

Barbara Annicchiarico

BBLM del Dipartimento del Tesoro

7 Giugno 2010

Motivation

- During the last three decades macroeconomic modelling has recorded deep changes both in methodological and theoretical aspects.
- Basic DSGE models capture elements of the New Keynesian paradigm, of the New Classical school and of the real business cycle approach (RBC), with several features of apparently irreconcilable traditions of macroeconomic thought, reflecting the emergence of a new approach to the study of macroeconomics, known as the New Neoclassical Synthesis.
- Large scale DSGE models have found their way to policy institutions who make policy analysis and forecasts. Bank of Canada (ToTEM), Bank of England (BEQM), European Central Bank (NAWM), Norges Bank (NEMO), Sveriges Riksbank (RAMSES), the US Federal Reserve (SIGMA), the IMF (GEM), the European Commission (QUEST III).

Motivation

- Advantages of this approach

Motivation

- Advantages of this approach
  - it provides many results of a textbook IS-LM model in a *fully dynamic coherent micro-founded* context (better understanding of the transmission mechanisms of policy interventions and of shocks);

Motivation

- Advantages of this approach
  - it provides many results of a textbook IS-LM model in a *fully dynamic coherent micro-founded* context (better understanding of the transmission mechanisms of policy interventions and of shocks);
  - it should be possible to escape the Lucas (1976) critique, contrary to the traditional macroeconometric models in which the estimated parameters are not invariant to policy shifts or to expected policy changes;

Motivation

- Advantages of this approach
  - it provides many results of a textbook IS-LM model in a *fully dynamic coherent micro-founded* context (better understanding of the transmission mechanisms of policy interventions and of shocks);
  - it should be possible to escape the Lucas (1976) critique, contrary to the traditional macroeconometric models in which the estimated parameters are not invariant to policy shifts or to expected policy changes;
  - thanks to the developments in computational techniques, DSGE modelling is a *quite* flexible technique;

Motivation

- Advantages of this approach
  - it provides many results of a textbook IS-LM model in a *fully dynamic coherent micro-founded* context (better understanding of the transmission mechanisms of policy interventions and of shocks);
  - it should be possible to escape the Lucas (1976) critique, contrary to the traditional macroeconometric models in which the estimated parameters are not invariant to policy shifts or to expected policy changes;
  - thanks to the developments in computational techniques, DSGE modelling is a *quite* flexible technique;
  - it sheds new light on the linkages among monetary and fiscal policy, inflation and the business cycle.

Motivation

- Advantages of this approach
  - it provides many results of a textbook IS-LM model in a *fully dynamic coherent micro-founded* context (better understanding of the transmission mechanisms of policy interventions and of shocks);
  - it should be possible to escape the Lucas (1976) critique, contrary to the traditional macroeconometric models in which the estimated parameters are not invariant to policy shifts or to expected policy changes;
  - thanks to the developments in computational techniques, DSGE modelling is a *quite* flexible technique;
  - it sheds new light on the linkages among monetary and fiscal policy, inflation and the business cycle.
- In terms of modelling approach to macroeconomics, *is this the best of all possible world* ? Of course not.... many shortcomings.... many inconsistencies.... many *ad hoc* assumptions... etc.

Objectives & Methodology

- The aim of these lectures is to show the main features of large scale DSGE models in order to understand their explanatory (and predictive?) power and their shortcomings.
- Starting from *simple* models, frictions will be introduced in a gradual manner.
- Contents:
    - Basic RBC model (I lecture)
    - RBC with frictions (labour taxes, habit, adjustment costs on investments and labour) (I lecture)
    - Basic New Keyensian model (II lecture)
    - New Keyensian model with price indexation (II lecture)
    - RBC + New Keynesian (building a medium scale DSGE model) (III lecture)
    - Problems with large scale models (III lecture)

Basic RBC Model

General Features

- Theory of fluctuations (persistence, output does not show a strong tendency to return to its long-run trend)
- Business cycles driven by technology shocks (real shocks)
- Focus on the real side of the economy
  - Quantities (aggregate production, consumption, employment, investments etc.)
  - Relative prices (real wage, real interest rate)
  - Classical dichotomy: money is a veil (nominal variables do not affect real variables)

Basic RBC Model

General Features

- Absence of frictions or imperfections and symmetry
  - Perfect competition in all markets
  - All prices adjust instantaneously
  - Rational expectations
  - No asymmetric information
  - The competitive equilibrium is Pareto optimal
  - Firms are identical and price takers
  - Infinitely lived identical price-taking households
- Seminal papers: Kydland and Prescott (1982), Nelson and Plosser (1982), Long and Plosser (1983), Prescott (1986).

The production function is a Cobb-Douglas

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$$

y_t production (real)
A_t stochastic technology process (technology innovation is *Hicks neutral*)

$$\begin{aligned} A_t &= A \exp z_t \\ z_t &= \rho_z z_{t-1} + \varepsilon_{z,t} \\ \varepsilon_{z,t} &\sim iid.N(0, \sigma_z^2) \end{aligned}$$

k_t capital stock
l_t labour
$A > 0, 0 < \rho_z < 1, 0 < \alpha < 1$

Firms hire workers at wage w_t and rent capital from the households at a rental rate r_t

Problem of the typical firm (max profits) (N.B. This is a static problem!)

$$\max_{\{k_t, l_t\}} \left(\underbrace{A_t k_t^{\alpha} l_t^{1-\alpha}}_{y_t} - w_t l_t - r_t k_t \right)$$

By profit maximization:

$$\underbrace{\alpha A_t k_t^{\alpha-1} l_t^{1-\alpha}}_{PMK} = r_t \qquad \underbrace{(1-\alpha) A_t k_t^{\alpha} l_t^{-\alpha}}_{PML} = w_t$$

Firms hire labour and capital until the (real) wage rate is equal to marginal product of labour and (real) rental rate of capital is equal to marginal product of capital. Factor payments exhaust all output:

$$y_t = w_t l_t + r_t k_t$$

The representative household maximizes

$$\max_{\{c_t, l_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + V(1 - l_t)]$$

c_t consumption (real)
$1 - l_t$ leisure
β is the time discount factor
Households earn a wage w_t and own the firms, so they receive rents $r_t k_t$. The budget constraint is

$$c_t + k_{t+1} = w_t l_t - tax_t + r_t k_t + (1 - \delta) k_t \;\; \forall t > 0$$

δ rate of capital depreciation
tax_t lump-sum taxes
Remarks: k_t is a predetermined endogenous variable; w_t and r_t are taken as given (they depend on the tech process A_t)

Solve the household intertemporal problem. Define the Lagrangian function:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} [u(c_t) + V(1-l_t)] + \\ +\lambda_t \left[w_t l_t - tax_t + r_t k_t + (1-\delta) k_t - c_t - k_{t+1} \right] \end{array} \right\}$$

λ_t is the Lagrange multiplier.
At the optimum

$$\begin{aligned} u'(c_t) &= \lambda_t \\ V'(1-l_t) &= \lambda_t w_t \\ \lambda_t &= \beta E_t \lambda_{t+1} (r_{t+1} + 1 - \delta) \end{aligned}$$

Labour supply:

$$\frac{V'(1-l_t)}{u'(c_t)} = w_t$$

The marginal rate of substitution between consumption and leisure is equal to the wage.

The time path of consumption is described by the stochastic **Euler equation**:

$$u'(c_t) = \beta E_t u'(c_{t+1})(r_{t+1} + 1 - \delta)$$

Household equates the cost from saving one additional unit of today's consumption to the benefit of obtaining more consumption tomorrow

$\rightarrow$ consumption smoothing

Consumption depends upon expected future wealth as opposed to current income.

The transversality condition

$$\lim_{s\rightarrow\infty} \beta^s u'(c_{t+s}) k_{t+s} = 0$$

It provides an *extra* optimality condition for the consumer's intertemporal optimization problem: if the time horizon were $t+s$, then it would not be optimal to have any capital left at time $t+s$ (if consumed it would give a discounted utility of $\beta^s u'(c_{t+s}) k_{t+s}$ at time t).

Simple balanced-budget rule:

$$g_t = tax_t$$

g_t exogenous government spending subject to shocks

$$\begin{aligned} g_t &= g \exp(\gamma_t) \\ \gamma_t &= \rho_\gamma \gamma_{t-1} + \varepsilon_{\gamma,t} \end{aligned}$$

where $\varepsilon_{\gamma,t} \sim iid.N(0, \sigma_\gamma^2)$.
(Alternatively: $b_t = (1 + r_t)b_{t-1} + g_t - tax_t$ with $tax_t = tax_0 + \tau b_{t-1}$; here there's Ricardian Equivalence)
We assume

$$\begin{aligned} u(c_t) &= \log c_t \\ V(1 - l_t) &= \psi \log(1 - l_t) \end{aligned}$$

Given an initial level of capital k_0 and the exogenous processes governing g_t and A_t, a competitive equilibrium is characterized by the sequence $\{c_t, l_t, y_t, i_t, k_{t+1}, tax_t, w_t, r_t\}_{t=0}^{\infty}$ satisfying the following conditions.

The consumption Euler equation from the household's problem

$$\frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}} \left(r_{t+1} + 1 - \delta \right)$$

The labour supply equation

$$\psi c_t = w_t \left(1 - l_t \right)$$

The labour demand and the capital return

$$\begin{aligned} \left(1-\alpha\right) A_t k_t^{\alpha} l_t^{-\alpha} &= w_t \\ \alpha A_t k_t^{\alpha-1} l_t^{1-\alpha} &= r_t \end{aligned}$$

Basic RBC Model

Equilibrium

The production function

$$y_t = A_t k_t^{\alpha} l_t^{1-\alpha}$$

The capital accumulation equation

$$i_t = k_{t+1} - (1-\delta) k_t$$

The aggregate resource constraint of the economy

$$y_t = c_t + i_t + g_t$$

The government budget constraint

$$g_t = tax_t$$

Tech and public spending stochastic processes

$$A_t = A \exp z_t \qquad z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$$

$$g_t = g \exp \gamma_t \qquad \gamma_t = \rho_\gamma \gamma_{t-1} + \varepsilon_{\gamma,t}$$

The model does not have a *paper and pencil* solution.

- What to do:

The model does not have a *paper and pencil* solution.

- What to do:

1. Find the necessary equations characterizing the RE equilibrium - OK

The model does not have a *paper and pencil* solution.

- What to do:

1. Find the necessary equations characterizing the RE equilibrium - OK
2. Solve for the deterministic steady state - OK

The model does not have a *paper and pencil* solution.

- What to do:

1. Find the necessary equations characterizing the RE equilibrium - OK
2. Solve for the deterministic steady state - OK
3. Calibrate parameter values $(\alpha, \beta, \delta, \rho_z, \rho_\gamma)$ and critical ratios

The model does not have a *paper and pencil* solution.

- What to do:

1. Find the necessary equations characterizing the RE equilibrium - OK
2. Solve for the deterministic steady state - OK
3. Calibrate parameter values $(\alpha, \beta, \delta, \rho_z, \rho_\gamma)$ and critical ratios
4. Linearize the intra and intratemporal optimality conditions around the deterministic steady state (methods based on second (or higher)-order Taylor expansion available)

The model does not have a *paper and pencil* solution.

- What to do:

1. Find the necessary equations characterizing the RE equilibrium - OK
2. Solve for the deterministic steady state - OK
3. Calibrate parameter values $(\alpha, \beta, \delta, \rho_z, \rho_\gamma)$ and critical ratios
4. Linearize the intra and intratemporal optimality conditions around the deterministic steady state (methods based on second (or higher)-order Taylor expansion available)
5. Compute the policy functions

The model does not have a *paper and pencil* solution.

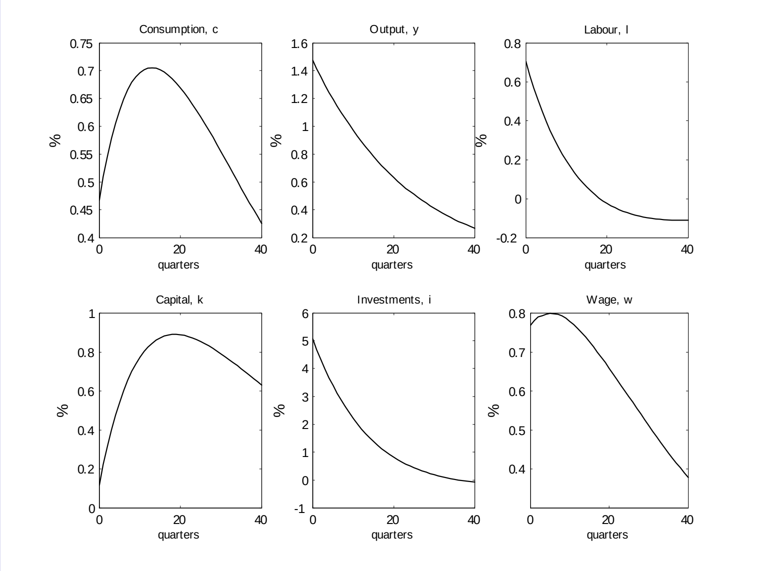
- What to do:

1. Find the necessary equations characterizing the RE equilibrium - OK
2. Solve for the deterministic steady state - OK
3. Calibrate parameter values $(\alpha, \beta, \delta, \rho_z, \rho_\gamma)$ and critical ratios
4. Linearize the intra and intratemporal optimality conditions around the deterministic steady state (methods based on second (or higher)-order Taylor expansion available)
5. Compute the policy functions
6. Analyze the model plotting impulse responses, computing moments and running stochastic simulations.

- Here we use **Dynare**, a pre-processor and a collection of MATLAB® (and GNU Octave) routines which solve non-linear models with forward looking variables using **perturbation methods**. For details see http://www.dynare.org and Collard and Juillard (2001a, 2001b).
- Basic idea of perturbation methods: formulate a general problem (our model), find a particular case that has a known solution (the deterministic steady state), use that particular case and its solution as a starting point for computing approximate solutions to the nearby problems (methods relying on Taylor series expansions, implicit function theorem see Judd 1998, Schmitt-Grohé and Uribe 2004).
- Dynare uses a Taylor approximation, up to third order, of the expectation functions.
- Here we undertake a second order perturbation of the model.

$\alpha = 0.33$	capital share
$\beta = 0.99$	discount factor
$\delta = 0.023$	depreciation rate
$\rho_Z = 0.95$	persistence of tech shock*
$\rho_\gamma = 0.95$	persistence of the public spending shock*
$Y = 1$	output
$L = 0.3$	employment

Effects of a Technology Shock (RBC)



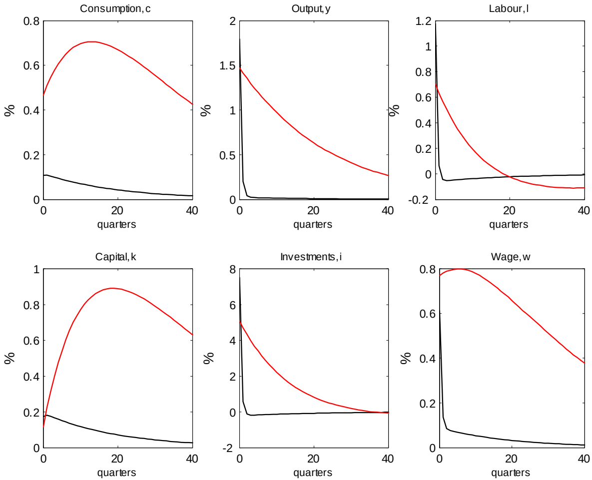
Effects of a Positive Technology Shock (RBC)

Propagation of the shock

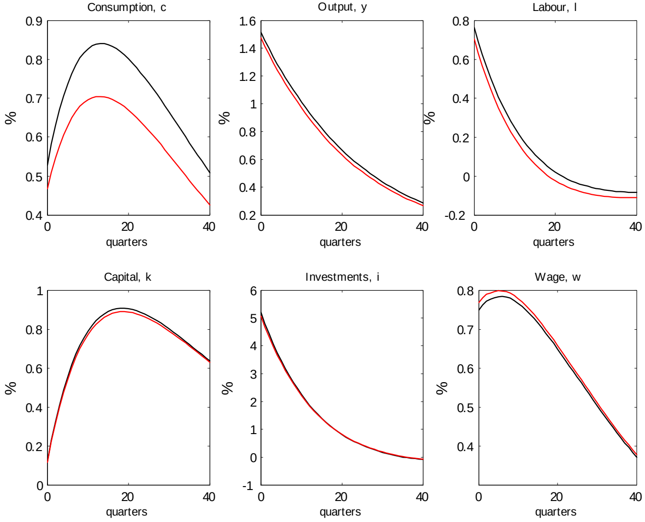
- Productivity $\uparrow \rightarrow$MPL $\uparrow \rightarrow$wage $\uparrow$
  - Substitution effect increases labour supply (prevails)
  - Income effect decreases labour supply
  - However a transitory productivity shock, which *temporarily* raises the real wage rate, increases labour supply today (agents work more today to be able to consume more in the future when the wage is expected to be lower)
- Productivity $\uparrow \rightarrow$MPK $\uparrow \rightarrow$rental rate of capital $\uparrow$
  - Substitution effect increases savings (prevails)
  - Income effect decreases savings
  - Consumption increases gradually (*consumption smoothing*)
  - Investments increases on impact (*the volatile component*)
  - Capital accumulates gradually and then slowly returns to its initial level
  - As a result y increases more than proportionally.

Effects of a Technology Shock (RBC) with Low Persistence

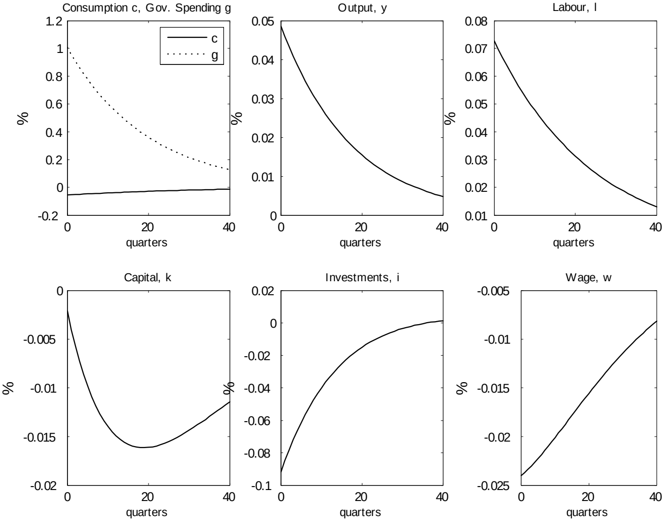
$\rho_Z = 0.1$



Effects of a Technology Shock (RBC) with IES=1.25



Effects of a Government Spending Shock (RBC)



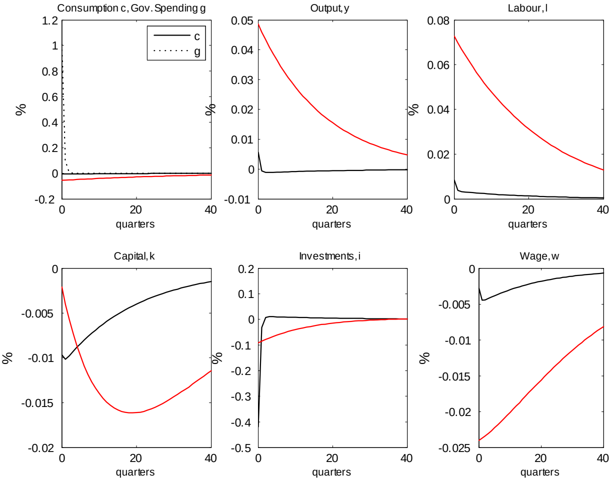
Effects of a Government Spending Shock (RBC)

Propagation of the shock

- Government spending $\uparrow$ $\rightarrow$taxes increase $\uparrow$ $\rightarrow$net wage income$\downarrow$
  - income effect (agents need to work more) $\rightarrow$employment increases then gradually returns to normal
  - consumption falls, but the rise in government supply is temporary, hence agents respond by decreasing their capital holdings (consumption smoothing)
  - the capital stock is only slightly affected the maximum impact after 17 quarters
- As a result y will increase less than proportionally.
- Remark: no comovement between c and g.

Effects of a Government Spending Shock (RBC) with Low Persistence

$\rho_\gamma = 0.10$



Adding Frictions to the Basic RBC Model

- Distortionary Taxes (Labour Taxes)
- External Habit
- Adjustment Costs on Investments
- Labour Adjustment Costs
- Other common frictions not considered here: e.g. indivisible labour *à la* Hansen 1985 (it is assumed that hours of work are fixed by firms and individuals simply decide whether or not to participate in the labour force), variable capacity utilization, taxes on capital and/or consumption.

As above... but now lump-sum taxation is not available.
Motivation: a large component of taxes is not lump-sum!
Now the budget constraint of the representative household is

$$c_t + k_{t+1} = \underbrace{(1-\tau_{t,l})\, w_t l_t}_{\text{wage income net of taxes}} + r_t k_t + (1-\delta) k_t$$

The new labour supply is

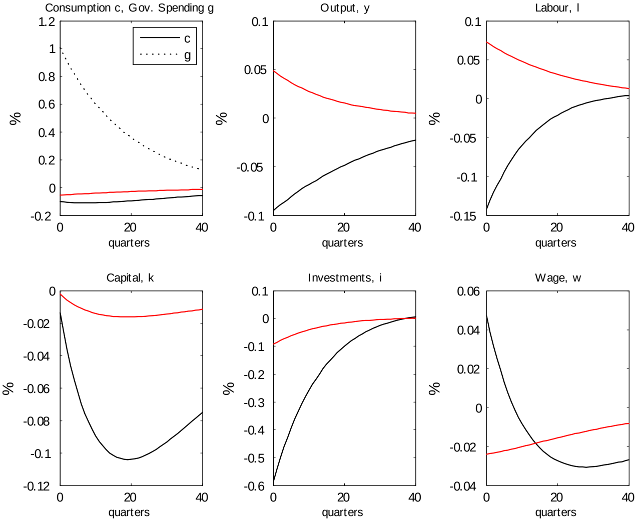
$$\frac{V'(1-l_t)}{u'(c_t)} = (1-\tau_{t,l})\, w_t$$

Pulic sector budget constraint is now

$$g_t = \tau_{t,l} w_t l_t$$

Set the initial level $\tau_{t,l} = 0.15$

Effects of a Government Spending Shock (RBC with Labour Taxes & RBC)



Effects of a Government Spending Shock (RBC with Labour Taxes)

Propagation of the shock

- Government spending $\uparrow$ $\rightarrow$labour tax rate $\uparrow$ $\rightarrow$net wage income$\downarrow$
  - income effect (agents would tend to work more)
  - substitution effect: the opportunity cost of leisure is now lower (agents would tend to work less)
  - consumption falls, but the rise in government supply is temporary, hence agents respond by decreasing their capital holdings (consumption smoothing)
  - the capital stock is now strongly affected
- As a result y will now decrease
- Remark: again no comovement between c and g.

RBC Model with External Habit

As above... but now the utility derived from consumption of the representative household i is

$$u(c_t^i - h_e c_{t-1})$$

- Habit persistence (the period utility function depends on a quasi-difference of consumption)
- $h_e \in (0, 1)$ = the intensity of habit formation.
- Here habits are external to the consumer: the stock of habit depends on the past value of aggregate consumption, c_{t-1} ('catching up with the Joneses', see Abel 1990).
- Under external habit an increase in the value of aggregate consumption will increase the marginal utility of consumption of each consumer i in the next period, inducing her to consume more.
- Motivation: in the data the response of consumption to expansionary shocks is hump-shaped (persistence).

RBC Model with External Habit

Under log specification of the utility of consumption and leisure the Euler equation is

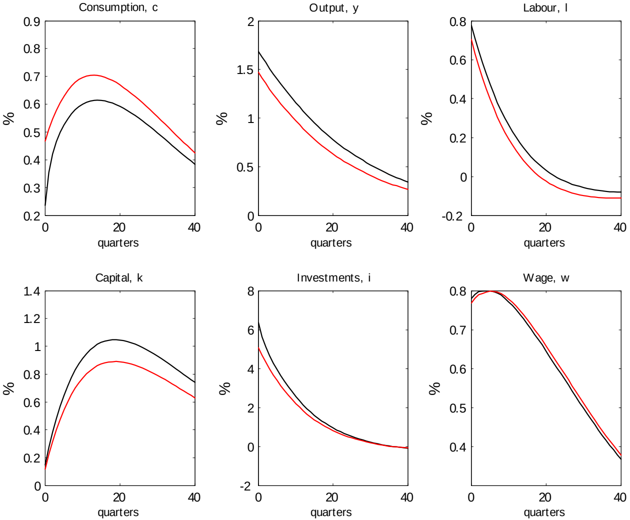
$$\frac{1}{c_t - h_e c_{t-1}} = \beta E_t \frac{1}{c_{t+1} - h_e c_t} \left(r_{t+1} + 1 - \delta \right)$$

the labour supply is now

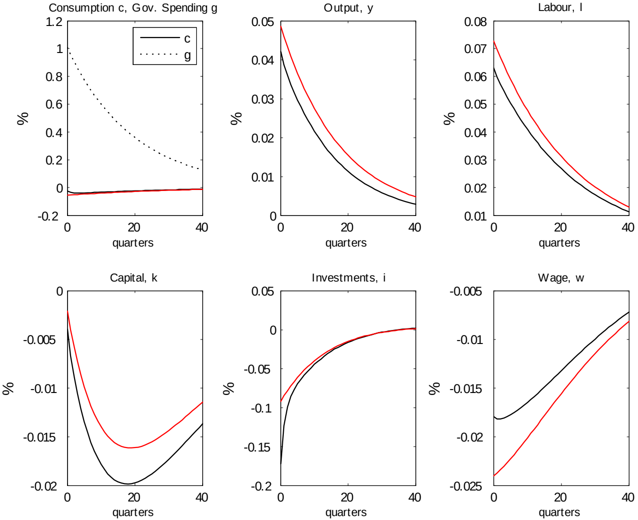
$$\psi \frac{1}{1 - l_t} = \frac{1}{c_t - h_e c_{t-1}} w_t$$

Set $h_e = 0.4$

Effects of a Technology Shock (RBC with External Habit & RBC)



Effects of a Government Spending Shock (RBC with habit & RBC)



RBC Model with Adjust. Costs on Investments

- **Motivation:** Simulations of standard RBC models produce too volatile investments. Convex capital installation costs make investment less volatile $\rightarrow$ more gradual dynamic adjustment in response to shocks.
- Why do we care? From the perspective of stabilizing policies and the impact of monetary policy investment is a key variable in the transmission mechanism.
- Many fiscal policy instruments affect the economy performance through the influence they have on capital accumulation.
- Here we assume that capital accumulation is costly (installation costs, disruption of productive activities, the need to retrain workers, the need to change the production process etc...).

RBC Model with Adjust. Costs on Investments

The new budget constraint of the household is

$$c_t + k_{t+1} = w_t l_t - tax_t + r_t k_t + (1-\delta)k - adj(i_t, k_t)$$

where

$$adj(i_t, k_t) = \frac{\phi_i}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t$$

Remark: in steady state $adj(i, k) = 0$ since $i = \delta k$.
The new Lagrangian is

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \log c_t + \psi \log(1 - l_t) \\ +\lambda_t \left[w_t l_t - tax_t + r_t k_t - i_t - c_t - \frac{\phi_i}{2}\left(\frac{i_t}{k_t} - \delta\right)^2 k_t \right] \\ +\xi_t \left[i_t + (1-\delta)k_t - k_{t+1} \right] \end{array} \right\}$$

RBC Model with Adjust. Costs on Investments

At the optimum:

$$\phi_i \left(\frac{i_t}{k_t} - \delta \right) = q_t - 1$$

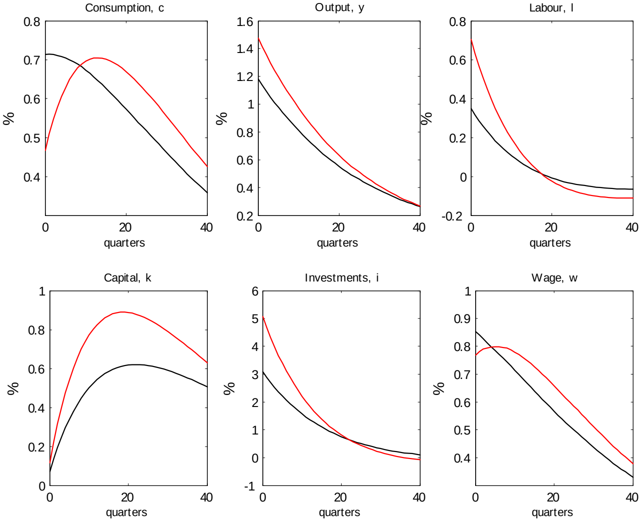
$$\underbrace{q_t \lambda_t}_{\xi_t} = \beta E_t \lambda_{t+1} r_{t+1} + \beta (1-\delta) E_t \underbrace{q_{t+1} \lambda_{+1}}_{\xi_{t+1}} - \beta E_t \lambda_{t+1} \frac{\partial adj(i_{t+1}, k_{t+1})}{\partial k_{t+1}}$$

where $q_t = \frac{\xi_t}{\lambda_t}$ is the Tobin's marginal q (it measures the expected marginal gains of more capital)

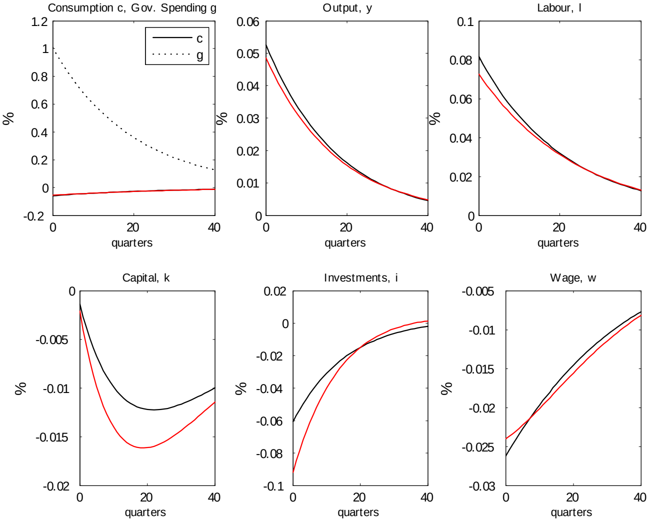
λ_t is the marginal benefit in terms of the utility of sacrificing a unit of current consumption in order to have an extra unit of investment, and so extra capital tomorrow

ξ_t is the marginal benefit in terms of utility of an extra unity of investments

Effects of a Technology Shock (RBC with Adjust. Costs on I & RBC)



Effects of a Government Spending Shock (RBC with Adjust. Costs on I & RBC)



RBC Model with Adjust. Costs on Labour

- **Motivation**: models of business cycle depend crucially on the operation of labor markets; attempts to forecast macroeconomic conditions often resort to consideration of observed movements in hours and employment to infer the state of economic activity; policy interventions in the labour market are numerous.
- Firms do face adjustment costs when changing the level of employment. Hiring and firing workers entail additional costs for firms (search and recruiting; training; explicit firing costs; variations in complementary activities; reorganization of production activities; capital accumulation etc.).
- As a result: the impact of adjustment costs on labour demand is to moderate changes in employment across the business cycle.

RBC Model with Adjust. Costs on Labour

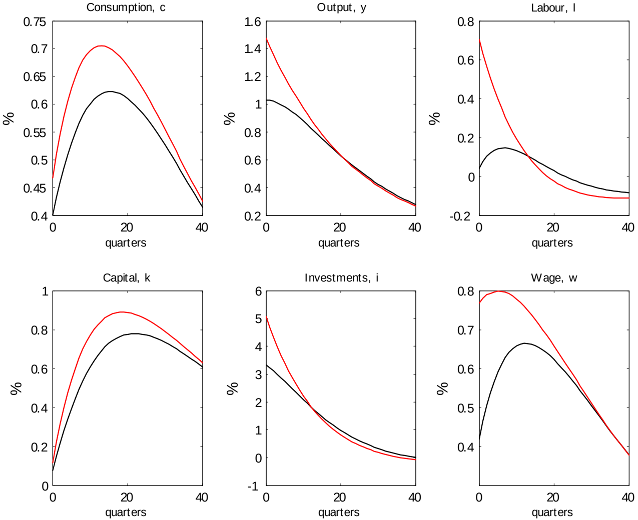
The representative firm's optimization problem is now intertemporal. The profit function is

$$\Pi_t = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left[\underbrace{A_t k_t^{\alpha} \left(l_t\right)^{1-\alpha}}_{y_t} - r_t k_t - w_t l_t - adj(l_t) \right]$$

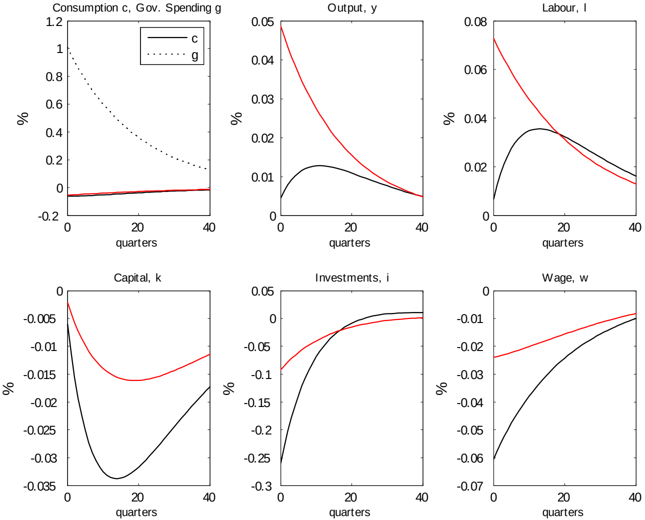
where $adj(l_t) = \frac{\phi_l}{2} \left(\frac{l_t}{l_{t-1}} - 1 \right)^2$.
convex cost of adjusting employment (ad hoc assumption: no quit rate, no discontinuities, no fixed costs).
The new demand of labour is

$$(1-\alpha) A_t k_t^{\alpha} l_t^{-\alpha} - \phi_l \left(\frac{l_t}{l_{t-1}} - 1 \right) \frac{1}{l_{t-1}} + \phi_l \beta \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{l_{t+1}}{l_t} - 1 \right) \frac{l_{t+1}}{l_t^2} = w_t$$

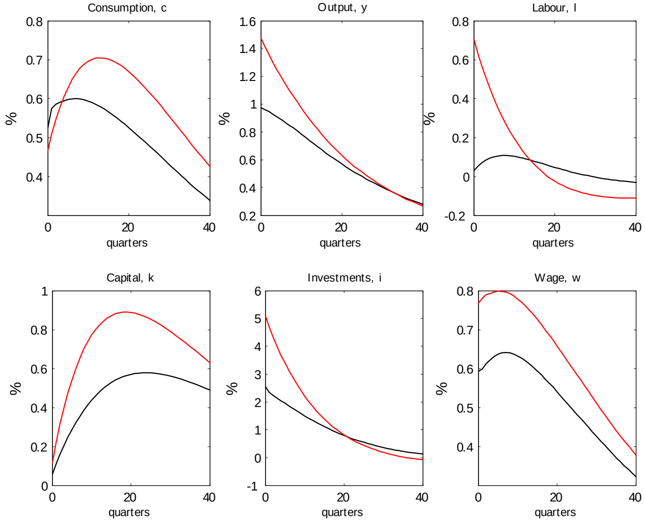
Effects of a Technology Shock (RBC with Adjust. Costs on L & RBC)



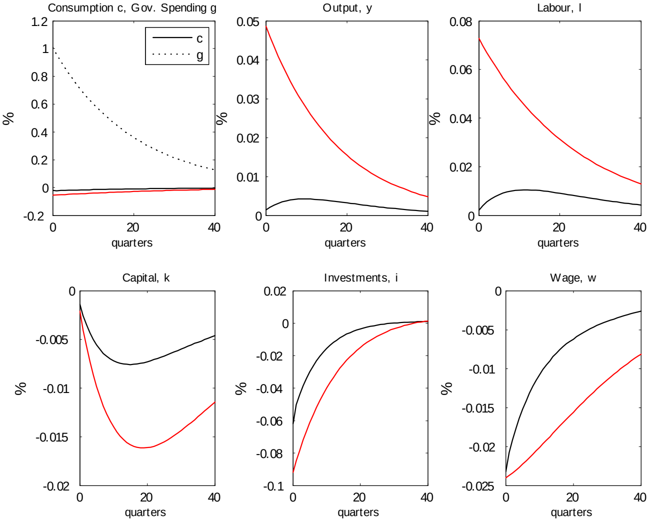
Effects of a Government Spending Shock (RBC with Adjust. Costs on L & RBC)



Effects of a Technology Shock (RBC with All Frictions & RBC)



Effects of a Government Spending Shock (RBC with All Frictions & RBC)



Conclusions

- The bulk of economic fluctuations could be interpreted as an equilibrium outcome resulting from the economy's response to real shocks. Cyclical fluctuations do not necessarily reflect inefficiencies (stabilization policies may not be desirable).
- The leading role of technology shocks as a source of economic fluctuations and of the persistence of output deviations from its trend (alternatively: model growth as endogenous!)
- Money is a only *veil*. RBC models are not suited for studying inflation, nominal interest rates and monetary policy. One-to-one relationship between prices and money aggregates.

References

Abel, A. B. (1990), Asset Prices under Habit Formation and Catching Up with the Joneses, American Economic Review, 80(2).

Adda, J., Cooper, R. (2003), Dynamic Economics, The MIT Press.

Collard, F., Juillard, M. (2001a), Accuracy of stochastic perturbation methods: The case of asset pricing models, Journal of Economic Dynamics and Control, 25(6-7).

Collard, F., Juillard, M. (2001b), A Higher-Order Taylor Expansion Approach to Simulation of Stochastic Forward-Looking Models with an Application to a Nonlinear Phillips Curve Model, Computational Economics, 17(2-3).

Galí, J. (2008), Monetary Policy, Inflation and the Business Cycle, Princeton University Press.

Judd, K. L. (1998), Numerical Methods in Economics, The MIT Press.

Kydland, F. E., Prescott, E. C., (1982), Time to Build and Aggregate Fluctuations,Econometrica, 50(6).

Long, J., Plosser, C., (1983), Real business cycles, Journal of Political Economy, 91.

McCandless, G. (2008), The ABCs of RBCs, Harvard University Press.

Nelson, C.R. Plosser, C.I., (1982),Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10.

Prescott, E.C., (1986), Theory Ahead of Business.Cycle Measurement, Carnegie-Rochester Conference Series on Public Policy, 25.

Romer, D. (2006), Advanced Macroeconomics, McGraw-Hill.

Schmitt-Grohe, S., Uribe, M.(, 2004), Solving dynamic general equilibrium models using a second-order approximation to the policy function, Journal of Economic Dynamics and Control, 28(4).

Wickens, M. (2008), Macroeconomic Theory, Princeton University Press.