The implementation of monetary and fiscal rules in the EMU: a welfare-based analysis

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Amedeo Argentiero(')

Abstract

This paper implements a methodology to evaluate the desiderability of monetary and fiscal rules within the context of the EMU using a DSGE model within a New Keynesian framework with sticky prices. The approach adopted is a welfare-based criterion that measures the welfare losses associated with these rules through a welfare loss function. Monetary policy follows a standard Taylor rule, whereas fiscal policy is made up of a countercyclical and debt-stabilizing public expenditure and of distortionary taxation on labor, dividends and interests on public bonds.

We find that: 1) when the economy is hit by a productivity shock the dynamic response of public debt is countercyclical and hence stabilizing; 2) in the presence of our monetary rule alone, domestic inflation variance falls by more than it would be when only the fiscal rules are present, whereas output gap smoothing is stronger with the fiscal rules in isolation than with the monetary rule alone; 3) the combination of our monetary rule and fiscal rules reduces welfare losses more than the same rules singly considered.

"The responsibility of the Fiscal Department in our imaginary state are derived from a multiplicity of objectives. For present purposes these are grouped under three headings: The use of fiscal instruments to (1) secure adjustments in the allocation of resources; (2) secure adjustments in the distribution of income and wealth; and (3) secure economic stabilization" (Musgrave (1959)).

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1 INTRODUCTION

The Stability and Growth Pact (henceforth SGP) with its budget rules represents a sort of ex-post fiscal coordination mechanism for EMU countries, where there is the "cohabitation" of one independent monetary policy and many fiscal policies. The aim of these fiscal rules is to stabilize public debt with respect to GDP, through the control of deficit with respect to GDP, both in the short term and in the medium term. In words, the SGP is founded on the idea that excessive deficits and high debts with respect to GDP are able to jeopardize the economic architecture of the EMU.

A large part of the literature (Buti et al. (1997), Melitz (2000), Wyplosz (2002), Gali and Perotti (2003)) has developed different contributions to understand whether the SGP has tied the EMU members' hands in pursuing the stabilization of the business cycles through the instruments of fiscal policy (i.e. taxation and public expenditure). Nevertheless, evidence is not univocal and covers a very short time span (ten years at most) for the construction of a representative sample in the EMU that incorporates different scenarios for the business cycle and the application of fiscal policy instruments to it.

In particular, Gali and Perotti (2003) in an empirical paper try to understand whether the Maastricht convergence criteria and the SGP requirements have weakened the stabilizing role of fiscal policy in EMU countries. The authors point out that fiscal policy in the EMU has become more countercyclical over time, following what appears to be a trend that affects other industrialized EMU and non-EMU countries as well; therefore, the SGP constraints would not represent an impediment on a stabilization path. Moreover, the decline in public investment, observed in the recent data, seems to follow a common tendency in other countries and starts before the implementation of the SGP. However, as Gali and Perotti state, real recessions in the period after-Maastricht have been quite rare and hence the available data are not so binding to announce a "failure" in the stabilizing activity of fiscal policy.

Furthermore, the reform of the SGP of 2005 aims to give a greater flexibility to the budget rules not in the thresholds for deficit and debt with respect to GDP, that remain unchanged at the levels of 3% and 60% respectively, but in taking into account the cyclical conditions of the economy whenever the deficit ceiling is exceeded. This is equivalent to let the instruments of fiscal policy stabilize the business cycle by ensuring a sustainability of public debt in the long term. In fact, the "new rules" regarding to fiscal policy objectives state that:
Beetsma and Debrun (2007) develop a theoretical model of fiscal policy trying to capture an important policy trade-off related with the reform of the SGP. In fact, although the "new rules" are able to enhance the role of fiscal policy in contrasting business cycle fluctuations so as to improve welfare, it is in practice necessary a strict discipline in distinguishing which kind of excessive deficits are really "acceptable" from those that are "unacceptable". In fact, the wide set of "any relevant factor", according to the authors, is able to reduce the role of judgement and weaken the enforcement power of the SGP in favor of an increasing politicization of the implementation of its rules.

The goal of our paper is to evaluate, within a DSGE model, the performances of two non-balanced budget fiscal rules, on public expenditure and taxation, that interact with a union-wide monetary policy rule, i.e. a standard Taylor rule. Public expenditure follows a countercyclical and debt-stabilizing rule, as implicitly required by the SGP from fiscal policy; taxation, instead, is given by the sum of tax revenues coming from lump-sum taxation, distortionary taxation on labor, on dividends and interests on public bonds issued by the government to finance its stock of debt.

The impulse response analysis undertaken in section 5 shows that tax revenues are procyclical and hence taxation works as an automatic stabilizer over the business cycle whenever the economy is hit by a productivity shock: i.e. taxation smooths the cyclical fluctuations of the economy around its long-run trend. In this way, both public expenditure and taxation pursue the objective of business cycle stabilization, the former ex-ante (by construction), because it is driven by public debt and production objectives, that are the steady state values, whereas the latter ex-post as an automatic response to the dynamics of the model.

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1 Therefore, taxation does not properly follow a rule with ex-ante objectives, but it is rather a collection of the total amount of tax revenues divided into their several components.
The theoretical assumptions and the simulated results, that identify a stabilizing role for fiscal policy tout court over the business cycle, are consistent with Musgrave’s theory of public finance (1959). This theory designs three purposes for fiscal policy: the provision for social goods, i.e. the allocation function of budget policy; the distribution of wealth among the citizens to equalize the incomes, i.e. the distribution function and the business cycle stabilization, i.e. the stabilization function.

In particular, according to the Musgravian theory, this last aim is pursued through the maintenance of a level of aggregate demand able to secure price-level stability and full employment. In fact, if involuntary unemployment prevails, an increase in the level of demand is needed through an adjustment of aggregate expenditure upwards so as to make the value of output produced at full employment. This role of aggregate demand support should belong to an increasing public expenditure and/or a reduction in taxation. Our analysis makes use of a microfounded theoretical New-Keynesian model with sticky prices à la Calvo (1983) applied to a currency union and a welfare loss function for each country belonging to the currency union to compute the consumers' losses in the presence of monetary and fiscal rules, following a large body of literature (Gali and Monacelli (2008), Beetsma and Jensen (2005), Ferrero (2009) among others). This approach has been implemented by a large literature on monetary policy rules, relying on a second-order approximation to the utility losses of the households caused by deviations of variables from their efficient allocation values (Rotemberg and Woodford (1999) and Gali (2008) among others). Di Giorgio and Nisticò (2008) evaluate, under alternative monetary regimes and in the presence of productivity shocks internationally transmitted across countries, the performances of fiscal deficit feedback rules with different degrees of fiscal discipline, defined as the response of fiscal rules to the existing stock of public debt, and the implications for net foreign assets and exchange rate dynamics. Their framework makes use, as ours, of a positive approach in studying the outcome of fiscal and monetary rules without deriving the optimal fiscal-monetary mix of policies. Our paper, instead, although does not derive the optimal fiscal and monetary regime, tries to measure the effects of the introduction of fiscal and monetary regimes through a welfare analysis. Also Ferrero (2009) uses a welfare-based approach to evaluate the desiderability of fiscal and monetary rules in a currency union. However, his analysis differs from our own for the presence of a rule on the real stock of public debt rather than on public consumption as in my own work and also for the use of a different welfare loss function. Indeed, Ferrero (2009) uses Benigno and Woodford's (2005) welfare loss function, that is able to take into account the presence of distortionary taxation and a positive stock of debt with corresponding steady state values different from the ones of the central planner’s solution. Hence, he derives optimal fiscal-monetary rules in a framework with distortions and compares this welfare-maximizing rules with flexible ones.

\[^2\text{In the fiscal rule for public expenditure all the variables involved (public expenditure, debt and production) are measured in log-deviation from the steady state values.}\]
In particular the author deals with a fiscal rule, that relates the real stock of public debt with output gap in a countercyclical way and a monetary rule that takes the form of a flexible inflation targeting together with a countercyclical monetary feedback to variations in economic activity. The interesting result found by Ferrero is that the welfare costs of balanced budget rules are at least one order of magnitude higher than the estimates of costs related to business cycle fluctuations.

The welfare loss function adopted in this paper, on the contrary, has the same structure as the one of Gali and Monacelli (2008): the arguments of this function are the domestic inflation squared, the output gap squared and the fiscal gap squared. The benchmark value of these variables against which we measure the losses is represented by a fully flexible prices equilibrium with lump-sum taxation able to finance public consumption, in the absence of public debt and with a monetary policy managed by the common central bank able to influence aggregate inflation and without real effects.

In particular, we compare three different scenarios: in the first one there is the only presence of our monetary rule with lump-sum taxation able to finance public consumption and in the absence of public debt; in the second scenario there are only fiscal rules and monetary policy is conducted through a simple “interest rate peg” to its long-run equilibrium value, whereas in the third scenario both fiscal and monetary rules are present. Here is an overview of the results:

- in the aftermath of a productivity shock, the dynamic response of real public debt is countercyclical and hence stabilizing;
- the presence of our monetary rule in isolation is able to generate a stronger decrease in domestic inflation variance than the one obtained under our fiscal rules in isolation;
- the presence of our fiscal rules in isolation generates an output gap smoothing stronger than the one obtained under our monetary rule in isolation;
- the combination of monetary and fiscal rules generates less welfare losses than in the case in which the rules are considered in isolation.

Hence, in the EMU the attainment of price stability should depend on the common monetary policy, whereas fiscal policy, institutionally decentralized at a country level, should be focused on output gap stabilization, that, in turn, can be reached through rules that ensure countercyclicality of fiscal policy, combined with debt stabilization, as required by the SGP.

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3 Fiscal gap is defined as the share of output used for public consumption less the amount of public expenditure to which the households give a weight in the utility function.
The paper is organized as follows. The model structure and its properties are set out in section 2. In Section 3, we derive the equilibrium market clearing conditions for the demand side of the market and for the supply side; in section 4, we discuss the calibration of the model parameters, whereas in section 5 we analyse, through impulse response analysis, the implications for the macro variables of a technology shock. In section 6, we conclude.

2 A CURRENCY UNION MODEL FOR FISCAL POLICY

We develop a closed currency union model, in the spirit of Gali and Monacelli (2008), made up of a continuum of small open economies, represented by the unit interval, so that the domestic policy decisions do not have any effect on the rest of the union.

Benigno (2004), Beetsma and Jensen (2005) and Ferrero (2009) among others build monetary union models by using a two-country scheme in which the dimension of each country has a specific role; nevertheless we consider our framework more suitable to give a realistic description of the inner structure of a monetary union like the EMU, made up of fifteen members (each one with an independent fiscal authority) with the prospective of a further enlargement. In fact and in line with a small country model, EMU countries are small relative to the union as a whole. Hence, each country’s policy decisions have very little impact on the other countries.

Indeed, this context, as a matter of principle, could be described by widening the existing one to incorporate fifteen countries, but such undertaking would render the resulting model intractable. In this model, all countries have identical agents’ preferences, technology and market structure; three agents are considered within each economy: the households, the firms and the government.

The firms are held by domestic households and the stock of public debt is financed through the emission of bonds bought by domestic households.
2.1 Households

All households living in the representative country \( i \) belonging to the monetary union aim to maximize an utility function defined over private consumption, \( C_i \), hours of work, \( N_i \) and public expenditure \( G_i \):

\[
E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_i, N_i, G_i \right)
\]

The consumption index \( C_i \) is defined as the Dixit-Stiglitz aggregator over the bundles of goods produced in country \( i \) and \( f \) respectively:

\[
C_i = \frac{(C_{i,t})^{1-m} (C_{f,t})^m}{(1-m)^{(1-m)} m^m}
\]

where \( C_{i,t} \) is a Dixit-Stiglitz aggregator defined over the continuum of differentiated goods produced in country \( i \) and \( j \in [0,1] \) denotes the type of good (within the set produced in country \( i \)). Each lot of goods is produced by a separate firm and no goods are produced in more than one country:

\[
C_{i,t} = \left( \int_0^1 C_{i,t}^j \left( j \right)^{(1-m)} dj \right)^{\frac{1}{1-m}}
\]

The aggregator \( C_{f,t} \), in turn, is an index of country \( i \)'s consumption of imported goods and represents an exogenous variable for country \( i \):

\[
C_{f,t} = \exp \int_0^{c_{f,t}} df
\]

where \( c_{f,t} = \log C_{f,t} \) is the log of an index of the goods consumed by country \( i \)'s households that are produced in country \( f \). This index is defined symmetrically to (3):

\[
C_{f,t} = \left( \int_0^{c_{f,t}} C_{f,t}^j \left( j \right)^{\frac{m-1}{m}} dj \right)^{\frac{1}{1-m}}
\]
Note that in the specification of composite consumption index above, \( m \in \{0; 1\} \) is the weight of imported goods in the utility of private consumption; we can think at \( m \) as an index of openness. The parameter \( \sigma > 1 \) is the elasticity of substitution across goods produced within one country.

The instantaneous utility function takes the following form, following Gali and Monacelli (2008):

\[
U^i \left( C_i^i, N_i^i, G_i^i \right) = (1 - \alpha) \log C_i^i + \alpha \log G_i^i - \frac{N_i^{1+\gamma}}{(1 + \gamma)}
\]  

(4)

where parameter \( \alpha \in \{0; 1\} \) represents the preference for public consumption \( G_i^i \).

All the prices are set in the common numeraire. The law of one price is assumed to hold, so that the price of each variety of goods is the same across countries. The implied overall consumption-based price index is:

\[
P^i_{C,t} = (P^i_t)^{1-m} (P^*_t)^m
\]  

(5)

where \( P^i_t = \left( \int_0^1 P^i_t(j)^{1-\sigma} \, dj \right)^{-\frac{1}{\sigma-1}} \) represents country \( i \)'s domestic price index (i.e., an index of prices of domestically produced goods), for all \( i \in \{0, 1\} \) and \( P^*_t = \exp \int_0^1 p^f_t \, df \) is the union-wide price index, that from the viewpoint of any individual country can be seen as the price index for imported goods. Symmetrically, the price index for the basket of goods imported from country \( f \) is defined as \( P^f_t = \left( \int_0^1 P^f_t(j)^{1-\sigma} \, dj \right)^{-\frac{1}{\sigma-1}} \). Note that the elasticities of domestic

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\(^1\text{In what follows, all the variables in small letters indicate the logarithms of the correspond}-\)

\(^1\text{ent variables in capital letters.}\)
price index and union-wide price index defined in (5) correspond to the relative weights of the respective goods in the consumption basket.

The maximization of (1) is subject to the following sequence of budget constraints (henceforth BC):

\[
\int_0^1 P_i^t(j)C_{i,t}^j(j) dj + \int_0^1 \int_0^1 P_i^t(j)C_{f,t}^j(j) dj df + D_i^t \leq (1 - \tau_i^t) W_i^t N_i^t + [1 + r_i^t (1 - \tau_i^t)] D_{i-1}^t + (1 - \tau_i^t) Y_{i-1}^t - Z_i^t
\]  

(6)

where \( P_i^t(j) \) is the price of the domestic goods (expressed in units of the single currency) and \( P_i^f(j) \) is the price of the imported goods. \( D_i^t \) is the nominal stock of public debt issued by the Government of country \( i \) held by domestic households in the form of bonds and whose law of motion is discussed later.

At the right member of the BC there are the resources available to domestic households at the beginning of the period \( t \): i) the nominal wages after labor taxation \( (1 - \tau_i^t) W_i^t N_i^t \), where \( W_i^t \) is the nominal wage and \( \tau_i^t \) is a distortionary wage tax levied by the government on labor; ii) the value of domestic bonds issued by the government of country \( i \) in period \( t - 1 \) after taxation on its yield, \( [1 + r_i^t (1 - \tau_i^t)] D_{i-1}^t \), where \( r_i^t \) is the nominal interest rate for the currency union, which follows a Taylor rule as explained in the next section and \( \tau_i^t \) is a distortionary tax levied by the government on bond yield; iii) the nominal profits after taxation, \( (1 - \tau_i^t) Y_{i-1}^t \), coming from the ownership of domestic firms purchased in \( t - 1 \) by the domestic households, where \( Y_{i-1}^t \) is the nominal profit of the representative firm of country \( i \). \( Z_i^t \) denotes lump-sum taxes, whose role will be discussed later. We assume that both public debt and the shares in monopolistic competitive firms can be only bought by domestic residents; hence, this is an additional dimension of incompleteness of financial markets together with the absence of contingent claims implicit in the model. Admittedly, this is a rather strong assumption, but it is introduced to simplify the theoretical structure of the model.

For each country, household’s consumption must be optimally allocated across all differentiated goods; expenditure on goods \( j \) is negatively related to the relative price of goods \( j \) and it satisfies the following first-order conditions:

\[
C_{i,t}^j(j) = \left( \frac{P_i^t(j)}{P_i^t} \right)^{-\sigma} C_{i,t}^j; C_{f,t}^j(j) = \left( \frac{P_i^f(j)}{P_f^t} \right)^{-\sigma} C_{f,t}^j
\]  

(7)

for all \( i, f, j \in [0, 1] \). It follows from the previous relationship that \( \int_0^1 P_i^t(j)C_{i,t}^j(j) dj = P_i^t C_{i,t}^j \) and \( \int_0^1 P_i^f(j)C_{f,t}^j(j) dj = P_i^f C_{f,t}^j \).

For country \( f \) symmetric conditions hold as for country \( i \); in particular, the consumer price index (CPI) is thus obtained:

\[
P_{c,t}^f = \left( P_i^f \right)^{1 - \tau f} \left( P_i^t \right)^{\tau f}
\]  

(8)
If we log-linearize and integrate both the members of the previous relationship over \( f \in [0, 1] \), we obtain the equality (9):

\[
P^*_c, f, t = \bar{p}_t^f
\]  

(9)

Moreover, the optimal allocation of expenditures for imported goods by country of origin implies:

\[
P^f_t C^i_{f, t} = P^*_t C^i_{f, t}
\]

(10)

for all \( f \in [0, 1] \).

Finally, combining all the previous results, we can express respectively the optimal allocation of expenditures between domestic and imported goods in country \( i \) and the total consumption expenditures by country \( i \)'s households in the following way:

\[
P^d_t C^i_{t, t} = (1 - \alpha) P^d_t C^i_{t, t}
\]

(11)

\[
P^f_t C^i_{f, t} = \alpha P^f_t C^i_{f, t}
\]

(12)

\[
P^d_t C^i_{t, t} + P^f_t C^i_{f, t} = P^*_t C^i_{t, t}
\]

(13)

Hence, the period budget constraint can be rewritten in a more compact way as:

\[
P^d_t C^i_{t, t} + D^i_{t, t} \leq [1 + \tau^i_t \left(1 - \tau^i_t\right)] D^i_{t-1} + (1 - \tau^i_t) \tau^i_{t-1} + (1 - \tau^i_n) W^i_t N^i_t - Z^i_t
\]

The remaining optimality condition for the control variables \((C^i_t, N^i_t)\) for the households are given by:

\[
C^i_t \cdot \left(N^i_t\right)^\gamma = (1 - \alpha) \frac{W^i_t \left(1 - \tau^i_n\right)}{P^*_t}
\]

(14)

For each household, the optimality condition for the amount of public debt to buy by the households in the form of domestic bonds leads to the following Euler equation:

\[
E_t \left[ \beta \left( \frac{P^d_{c, t} C^i_t}{P^*_{c, t+1} C^i_{t+1}} \right) \right] = Q_{t, t+1}
\]

(15)

where \( Q_{t, t+1} = R^i_{t+1} = \frac{1}{1 + \tau^i_n \left(1 - \tau^i_t\right)} \) is a discount factor after taxation on the interest rate. Note that (15) is not a standard version of the Euler equation: in fact the presence of taxation on interest payments on debt distorts the margin between consuming today and consuming tomorrow. This phenomenon is captured by a lower value of the term \( R^i_{t+1} \) with respect to the case of complete absence of any distortionary taxation.

For country \( f \) an Euler equation analogous to (15) holds:

\[
E_t \left[ \beta \left( \frac{C^f_t}{C^f_{t+1}} \right) \left( \frac{P^f_t}{P^*_{c, t+1}} \right) \right] = Q_{t, t+1}
\]

(16)
Combining (15) and (16), we obtain:

\[ C_t = \xi_t C_t^f \left( \frac{P_t^c}{P_t^c} \right) \]  

(17)

where \( \xi_t = \frac{P_t^f}{P_t^c} C_t^{\ell+1} \) is a constant which depends on initial conditions regarding relative net asset positions; if we suppose zero net foreign asset holdings for all countries together with the hypothesis of an ex-ante identical environment, we have the case in which \( \xi_t = \xi = 1 \) for all \( \ell \in [0, 1] \), i.e.:

\[ C_t^f = C_t^f \left( \frac{P_t^c}{P_t^c} \right) \]  

(18)

Moreover, in each country government’s assets are also subject to the following transversality condition:

\[ \lim_{T \to \infty} E_t \{ Q_{t,T+1} D_{T+1}^f \} = 0 \]  

(19)

\[ \lim_{T \to \infty} E_t \{ Q_{t,T+1} D_{T+1}^f \} = 0 \]  

(20)

In the next subsection, we describe the behavior of the central bank, that determines the monetary policy for the currency area, using the short-term nominal interest rate as its main instrument, but, before this step, we define the domestic CPI gross rate of inflation and the union-wide gross rate of inflation, respectively as:

\[ \Pi_{t,t}^i = \frac{P_{t,t}^n}{P_{t,t-1}^n} \]  

(21)

\[ \Pi_t^* = \frac{P_t^*}{P_{t-1}^*} \]  

(22)
2.2 Interest rate and monetary policy

The Central bank sets the short-term nominal interest rate \( r^*_t \) for the currency union as a linear function of the union-wide current inflation \( \pi^*_t \) and the union-wide output gap, defined here as the deviation of output \( y_t^\circ \) from \( y_t^\circ^o \), i.e. its level under fully flexible price value. It is typically assumed that the coefficient on inflation \( \phi_{\pi} \) is greater than one\(^5\), which implies that the central bank raises the nominal interest rate more than one-for-one in response to an increase in inflation.

We therefore postulate the following Taylor rule:

\[
r^*_t = \bar{r}^* + \phi_{\pi} (\pi^*_t) + \phi_{y} (y_t^* - y_t^*^o) \tag{23}
\]

The quantity \( \bar{r}^* \) is the steady-state equilibrium real interest rate, that can be derived from the Euler equation (15):

\[
\left[ \beta \left( 1 + \bar{r}^* (1 - \tau_k^u) \right) \right] = 1 \tag{24}
\]

\[
\bar{r}^* = \left( \frac{1}{\beta} - 1 \right) \frac{1}{(1 - \tau_k^u)} \tag{25}
\]

\(^5\) This condition is in line with the findings of Bullard and Mitra (2001) and satisfies the so-called "Taylor principle".
2.3 Firms

Each country has a continuum of firms represented by the interval \( j \in [0, 1] \). Each firm produces differentiated goods with a linear technology:

\[
Y_t^i(j) = A_t^i N_t^i(j)
\]  

(26)

where \( A_t^i \) is a country-specific aggregate technology index, whose law of motion follows an AR(1) process (in logs):

\[
a_t^i = \rho a_{i-1} + \varepsilon_{at}^i
\]

(27)

and \( \rho \in \{0,1\} \).

The assumption of linear technology implies that real marginal costs are given by:

\[
mc_t^i = \log (1 - s^i) + w_t^i - p_t^i - a_t^i
\]

where \( s^i \) is a constant employment subsidy with the role to offset firms’ market power represented by the monopolistic competition. This subsidy is completely financed by the lump-sum taxes \( Z_t^i \) introduced earlier, as in the model of Gali and Monacelli (2008).

We assume a staggered price setting à la Calvo (1983). As well described in Gali (2008), there is a number \( 1 - \theta \) of (randomly selected) firms, which sets new prices each period, with an individual firm’s probability of reoptimizing in any given period being independent of the time elapsed since its last price resetting. Hence, the parameter \( \theta \) is an index of stickiness. The aggregate price dynamics is described by the following equation:

\[
\Pi_t^i(1-\sigma) = \theta + (1-\theta) \left( \frac{P_t^R}{P_{t-1}^R} \right)^{1-\sigma}
\]

(28)

where \( \Pi_t^i = \frac{P_t^i}{P_{t-1}^i} \) is the gross inflation rate and \( P_t^R \) is the price set in period \( t \) by firms reoptimizing their price in that period. A firm reoptimizing in period \( t \) chooses a price \( P_t^R \) that maximizes the current market value of the profits \( \Upsilon_t^i \) along the horizon while that price remains in place.

Thus, the firms solve the following problem:

\[
\max_{P_t^R} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{i(t+k)}^i \left( P_t^R \Upsilon_t^i - \Psi_{t+k}^i \left( Y_t^i \right) \right) \right\}
\]

(29)
subject to the sequence of demand constraints

$$Y^i_{t+k|t} = \left( \frac{P^{iR}_{t+k}}{P^i_{t+k}} \right)^{-\sigma} C^i_{t+k}$$

for \(k = 0, 1, 2, \ldots\) and where \(Q^{i}_{t+k} = \frac{P^{iR}_{t+k}}{P^i_{t+k}} = \beta^k (C^i_{t+k}/C^i_t) \left( P^i_t / P^i_{t+k} \right) \) is the discount factor, \(\Psi^i_t(\cdot)\) is the cost function of the firm, whereas \(Y^i_{t+k|t}\) represents output in period \(t+k\) for a firm resetting its price in period \(t\). Next, the first order condition associated with the problem (29) is given by:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q^{i}_{t+k|t} Y^i_{t+k|t} \left( P^{iR}_{t+k} - M\psi^i_{t+k|t} \right) \right\} = 0$$

where \(\psi^i_{t+k|t} = \Psi^i_t \left( Y^i_{t+k|t} \right)\) indicates the nominal marginal cost in period \(t+k\) for a firm resetting its price in period \(t\) and \(M = \frac{c}{\sigma-1}\) is the desired markup in the absence of constraints on the frequency of price adjustment\(^5\). Note that in the absence of price rigidities \((\theta = 0)\) the previous condition collapses to the optimal price setting condition under flexible prices:

$$P^{iR}_{t+k} = M\psi^i_{t+k|t}$$

Moreover, in this particular case, by setting \(s^i = \frac{1}{\sigma}\) and substituting this value and the definition of nominal marginal costs into (32), an optimal market allocation, that is able to completely eliminate the consequences of monopolistic competition, can be reached. In fact, if \(s^i = \frac{1}{\sigma}\), expression (32) turns into the optimality condition of perfect competition, according to which the price should be equal to the marginal cost.

Then, we divide both the members of (31) by \(P^i_{t-1}\):

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q^{i}_{t+k|t} Y^i_{t+k|t} \left( \frac{P^{iR}_{t+k}}{P^i_{t-1}} - M \Psi^i_{t+k|t} \Pi^i_{t-1,t+k} \right) \right\} = 0$$

where \(MC^i_{t+k|t} = \frac{\psi^i_{t+k|t}}{P^i_{t+k}}\) is the real marginal cost in period \(t+k\) for firms whose last price set is in period \(t\) and, finally, we log-linearize the optimal price setting condition (33) around the zero inflation steady state with a first-order Taylor expansion:

$$p^{iR}_{t+k} - p^i_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[ \tilde{mc}^{i}_{t+k|t} + (p^i_{t+k} - p^i_{t-1}) \right]$$

where \(\tilde{mc}^{i}_{t+k|t} = mc^{i}_{t+k|t} - \overline{mc}\) is the log-deviation of marginal cost from its steady state value.

---

\(^5\) The analytical derivation is reported in the Appendix.
The optimal price setting strategy for the typical firm resetting its price in period \( t \) can be derived from (34), after some algebra:

\[
p_i^{IR} = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[ mc_{i+k|t} + p_{i+k} \right]
\]

(35)

with \( \mu = \log \frac{\sigma}{\sigma - 1} \), that represents the optimal markup in the absence of constraints on the frequency of price adjustment (\( \theta = 0 \)). Hence, the price setting rule for the firms resetting their prices is represented by a charge over the optimal markup in the presence of fully flexible prices, given by a weighted average of their current and expected nominal marginal costs, with the weights being proportional to the probability of the price remaining effective (\( \theta \)). In a zero-inflation steady state equilibrium and in the absence of price stickiness for all the firms (\( \theta = 0 \)), the previous expression collapses to:

\[
\bar{p} = \mu + 1
\]

(36)

Note that, under the hypothesis of constant returns to scale, implicit in the production function of our model, the marginal cost is independent from the level of production, i.e. \( mc_{i+k|t} = mc_{i+k} \) and, hence, common across firms; so, the expression (38) can be rewritten in the following way:

\[
p_i^{IR} - p_{i-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[ mc_{i+k|t} \right] + \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left[ \pi_{i+k|t} \right]
\]

(37)

Moreover, the equation (37) can be expressed as the following difference expression:

\[
p_i^{IR} - p_{i-1} = \beta\theta E_t \left[ p_i^{IR} - p_i \right] + (1 - \beta\theta) \bar{mc}_i + \pi_i
\]

(38)

and combined with (28) in a log-linear form in order to obtain the domestic inflation equation:

\[
\pi_i = \beta E_t \left[ \pi_{i+1} \right] + \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \bar{mc}_i
\]

(39)

The previous expression states that the current value of domestic inflation is positively related to the discounted expected value of the inflation of one period ahead and to the log-deviation of real marginal cost according to the degree of price stickiness captured by the parameter \( \theta \).
2.4 Government

Following the same structure of private consumption, country \( i \)'s public consumption index is given by

\[
G^i_t = \left( \int_0^1 G^i_t(j)^{\frac{\sigma-1}{\sigma}} dj \right)^{-\frac{1}{\sigma-1}}
\]  

(40)

where \( G^i_t(j) \) represents the quantity of domestic goods purchased by the government. In line with the empirical evidence (Trionfetti, 2000), we assume that government purchases are fully oriented towards domestic goods. Each government chooses optimally the composition of a Dixit-Stiglitz aggregator over all goods produced in its own country to minimize expenditure, yielding a structure of demand schedules analogous to those of private consumption:

\[
G^i_t(j) = \left( \frac{P^i_t(j)}{P^i_t} \right)^{-\sigma} G^i_t
\]  

(41)

Nevertheless, we want to focus our attention on the allocation of the aggregate level of primary public expenditure and we assume that government consumption is related to its lagged level, to the lagged real stock of public debt and to the real output according to the following rule

\[
\tilde{g}^i_t = \zeta^i \tilde{g}^i_{t-1} - \kappa^i \tilde{d}^i_{R,t-1} - \chi^i \tilde{y}^i_t
\]  

(42)

where \( \tilde{g}^i_t, \tilde{d}^i_{R,t-1} \) and \( \tilde{y}^i_t \) are the log-deviations from their corresponding steady state values. Moreover,

\[
\tilde{d}^i_{R,t-1} = d^i_{t-1} - \tilde{p}^i_{t-1}
\]

indicates the log-deviation from steady state of real public debt, the parameter \( \zeta^i \) indicates the degree of inertia in public expenditure, the parameter \( \kappa^i \) measures the magnitude of the feedback effect of debt and the coefficient \( \chi^i \) indicates the countercyclicality of public consumption.

Importantly, the rule (42) is not a model-based rule, but it simply tries to capture both the phenomenon of debt stabilization, as implicitly required by the SGP criteria, and the business cycle stabilization, that is an important objective of fiscal policy tout court. In fact,
"under conditions of depression, an increase in expenditure ordinarily leads to an increase in real output and employment" (Musgrave (1959)).

If, on the contrary,

"let us assume a situation of more or less full employment, so that the supply of total output is quite inelastic. An increase in the level of aggregate demand now produces inflation. Fiscal adjustment must be made to prevent a potential increase in the level of expenditure from coming about. The required adjustments are the inverse of those used in the depression case" (Musgrave (1959)).

The presence of a feedback on debt component in a spending rule has been also adopted for example by Kirsanova and Wren-Lewis (2007); these authors examine the impact of different degrees of feedback on debt for public expenditure in an economy with nominal rigidities where monetary policy is optimal; using a welfare function, they find the optimal level of fiscal feedback, which represents a threshold above which optimal monetary policy becomes less active and fiscal feedback does stabilize inflationary shocks, but with a welfare reduction. On the other hand, below this cut-off value, monetary policy becomes strongly passive with a deterioration in welfare.

Schmitt-Grohé and Uribe (2007) use a rule similar to (42) for taxation, i.e. the out-of-steady-state level of taxation is an increasing function of the lagged out-of-steady-state level of public liabilities, together with a monetary rule whereby the change in the nominal interest rate is set as a function of its own lag, lagged output growth, and lagged deviations of inflation from target. The authors maximize a welfare function in the presence of these rules and compare this framework with the Ramsey optimal policy: they find that interest rate rules with a positive response to output can lead to significant welfare losses, whereas optimal fiscal policy is passive. The optimal monetary and fiscal rule combination is able to attain the same level of welfare as the Ramsey optimal policy.
Muscatelli et al. (2004), in an empirical evaluation of monetary-fiscal interactions, estimate a New Keynesian dynamic general equilibrium model with the presence of monetary and fiscal rules. The former is based on a forward-looking Taylor rule specification, whereas the latter is based on a spending rule and on a taxation rule; both of them are built so that the variables are allowed to respond to output gap, to the ratio between the lagged budget deficit on GDP and to a persistence component, as for (42).

Recently, Cokiago et al. (2008) in a two-country New Keynesian DSGE model, incorporating non Ricardian consumers, in order to analyse the stabilizing role of national fiscal policies in a currency union, build a spending rule very similar to (42) with a feedback on debt term to explicitly take into account the SGP criteria on debt.

Our rule on taxation is such that real tax revenues are given by collecting lump sum taxation, distortionary taxation on labor, on domestic dividends and interests on bonds:

$$
\tau_{it} = \frac{Z_t}{P_t} + \tau_{it} \cdot \frac{W_t}{P_t} \cdot N_t + \tau_{it} \cdot D_{it} \cdot \frac{T_t}{P_t} \tag{43}
$$

Hence, real taxation is increasing in the hours worked and real wages, in domestic real stock of public debt and real amount of profits.

The law of motion of public debt is described by the following equation:

$$
D_{t-1} (1 + \tau_{it}^*) + P_t^i G_t^i - T_t^i = D_t^i \tag{44}
$$

that states that the stock of public debt in each period is equal to the present value of the past stock of public debt increased by the primary deficit, given by the difference between public expenditure and taxation ($P_t^i G_t^i - T_t^i$).
3 EQUILIBRIUM DYNAMICS

3.1 Aggregate Demand and Supply side

The market clearing conditions for the goods \( j \) in country \( i \) can be expressed in the following way:

\[
Y^i_t(j) = C^i_{i,t}(j) + \int_0^1 C^f_{i,t}(j) df + G^i_t(j)
\]

(45)

The previous relationship states that domestic production of good \( j \) can be allocated to domestic consumption, to foreign consumption (i.e. exports) and to public consumption. Then, following Gali and Monacelli (2008) and using the definitions of \( C^i_{i,t}(j), C^f_{i,t}(j) \) and \( G^i_t(j) \), we obtain:

\[
Y^i_t(j) = \left( \frac{P^i_t(j)}{P^i_t} \right)^{-\sigma} \left[ \frac{(1 - m) P^i_{c,t} C^i_t}{P^i_t} + m \int_0^1 C^f_t \left( \frac{P^i_{c,t}}{P^i_t} \right) df + G^i_t \right]
\]

(46)

\[
Y^i_t(j) = \left( \frac{P^i_t(j)}{P^i_t} \right)^{-\sigma} \left[ \frac{1}{P^i_t} \left( (1 - m) P^i_{c,t} C^i_t + m P^i_t C^i_t + G^i_t \right) \right]
\]

where

\[
C^i_t = \int_0^1 C^f_t df
\]

If we plug the previous relationship into (18), we are able to express \( C^i_t \) as a function of the domestic consumption:

\[
Y^i_t(j) = \left( \frac{P^i_t(j)}{P^i_t} \right)^{-\sigma} \left[ \frac{1}{P^i_t} \left( (1 - m) P^i_{c,t} C^i_t + m C^i_t P^i_t \right) + G^i_t \right]
\]

\[
Y^i_t(j) = \left( \frac{P^i_t(j)}{P^i_t} \right)^{-\sigma} \left[ \frac{P^i_{c,t}}{P^i_t} \left( C^i_t \right) + G^i_t \right]
\]

(47)

Finally, by plugging (47) into the definition of the aggregate output index for country \( i \), we obtain an expression of the aggregate domestic output:
\[ Y_t^i = \left[ \frac{P_{c,t}^i}{P_t^i} (C_t^i) + G_t^i \right] \]  

(48)

The term \( \log (Y_t^i - G_t^i) \) can be expressed in a first-order Taylor expansion about the steady state by the next expression\(^7\):

\[ \log (Y_t^i - G_t^i) = \log \left( (1 - \chi_t^i) \bar{Y}^i \right) + \frac{1}{1 - \chi_t^i} \left( \hat{y}_t^i - \chi_t^i \hat{g}_t^i \right) \]  

(49)

where \( \chi_t^i = \frac{\bar{y}_t^i}{\bar{Y}^i} \) is the steady state government spending share (see Galí and Monacelli (2008)).

From (49), rewriting (48) in a log-deviation from the steady state values and recalling the definition of the out-of-steady-state public expenditure, we are able to build the demand side of this economy:

\[ \hat{y}_t^i = \left[ (1 - \chi_t^i) \left( \hat{c}_t^i + (p_{c,t}^i - p_t^i) \right) + \frac{\chi_t^i}{1 + \chi_t^i} \left( \psi_t^i \hat{g}_{t-1}^i - \kappa_t^i \hat{d}_{R,t-1}^i \right) \right] \]  

(50)

The previous equation establishes that domestic output is positively related to domestic consumption, to the terms of trade and to the lagged real public expenditure, whereas output is decreasing in the lagged real stock of public debt. The negative relationship between domestic output and the lagged real stock of public debt represents the amount of resources withdrawn from public consumption in order to stabilize the real stock of public debt, consistently with the SGP budget rules.

Furthermore, the higher is the steady state government spending share, the lower is the weight of private consumption in the determination of domestic output due to the crowding out effect of public expenditure.

To build the supply side of this economy, first we have to rewrite the dynamics of domestic inflation:

\[^{7}\text{The analytical derivation is reported in the Appendix.}\]
\[ \pi_t^i = \beta \mathbb{E}_t \left[ \pi_{t+1}^i \right] + \frac{(1-\theta)(1-\beta\theta)}{\theta} \tilde{mc}_t^i \]  

(51)  

and recall the definition of real marginal cost (in logs):

\[ mc_t^i = \log (1-s^i) + w_t^i - p_t^i - a_t^i \]  

(52)  

Then we add and subtract to the right side of (52) the quantity \( p_{c,t}^i \):

\[ mc_t^i = \log (1-s^i) + (w_t^i - p_{c,t}^i) + (p_{c,t}^i - p_t^i) - a_t^i \]  

and combining the resulting expression with the previous results we obtain the next relationship:

\[ mc_t^i = \log (1-s^i) + c_t^i + \gamma n_t^i - \log (1-\alpha) - \log (1-\tau_n^i) + (p_{c,t}^i - p_t^i) - a_t^i \]  

(53)  

Note that, according to (48), the terms of trade \( (p_{c,t}^i - p_t^i) \) are equal to \( \log (Y_t^i - G_t^i) - c_t^i \), i.e.:

\[ mc_t^i = \log (1-s^i) + \gamma n_t^i - \log (1-\alpha) - \log (1-\tau_n^i) + \log (Y_t^i - G_t^i) - a_t^i \]  

(54)

Following Gall and Monacelli (2008), the combination of (54) and (49) leads to a definition of real marginal cost expressed in log-deviation from the steady state value\(^8\):

\[ \tilde{mc}_t^i = \left( \frac{1}{1 - \gamma} + \gamma \right) \tilde{y}_t^i - \frac{\gamma \bar{y}_t^i}{1 - \gamma} \tilde{g}_t^i - (1 + \gamma) \tilde{a}_t^i \]  

(55)

\(^8\)For this result we use the property that in a symmetric steady state the steady state value of the terms of trade is equal to one and so \( \bar{p}_c^i - \bar{p}_t^i = 0 \)
Given the level of output, an increase in government spending crowds out domestic consumption and generates a real appreciation: both these pressures lead to a reduction in real marginal cost, whose dimension is measured by the parameter $\kappa_i$.

Finally, by plugging (55) into (51), we are able to derive the new Keynesian Phillips curve for the domestic economy:

$$\pi_t^i = \beta E_t \left[ \pi_{t+1}^i \right] + \frac{(1 - \theta) (1 - \beta \theta)}{\theta} \left( \frac{1}{1 - \kappa_i} + \gamma \right) \dot{y}_t^i - \frac{\kappa_i}{1 - \kappa_i} \dot{y}_t^i - (1 + \gamma) \dot{a}_t^i$$

and, by integrating (56) over $i \in [0,1]$, we are able to obtain the corresponding new Keynesian Phillips curve for the union as a whole:

$$\pi_t^* = \beta E_t \left[ \pi_{t+1}^* \right] + \frac{(1 - \theta) (1 - \beta \theta)}{\theta} \left( \frac{1}{1 - \kappa_i} + \gamma \right) \ddot{y}_t^* - \frac{\kappa_i}{1 - \kappa_i} \ddot{y}_t^* - (1 + \gamma) \ddot{a}_t^*$$

where

$$Y_t^* = C_t^* + G_t^*$$

$$\alpha_t^* = \int_0^1 a_t d\tau$$

and all the other union-wide variables are defined in a symmetric way with respect to the country-specific one (see Gali and Monacelli (2008)).
3.2 The Efficient Allocation under Flexible Prices

Before describing the dynamic equilibrium conditions in the presence of nominal rigidities, we need to derive an expression for the flexible-price output $Y_i^{\text{in}}$, i.e. the natural level of output, in order to define a measure of the output-gap of each member’s economy and then the one of the whole monetary union. Note that, due to the presence of distortionary taxation, the only way to calculate the natural level of output is the solution of the decentralized economy under flexible prices, because in this case, the equilibrium derived by the solution of the social planner’s problem would not be supported in the decentralized context, as done by Gali and Monacelli (2008). In the absence of constraints on the frequency of price adjustment, the price setting rule follows the equation (32), i.e. each firm charges the price as a markup over the nominal marginal cost and the value of markup is optimal and equal to $M = \frac{\sigma}{\sigma - 1}$ and $mc = \log \left( \frac{\sigma}{\sigma - 1} \right)$. The procedure followed to characterize and derive the fully flexible price output $Y_i^{\text{in}}$ is given by the solution of the decentralized economy, as before, with a null value for the price-stickiness parameter $\bar{\theta}$, i.e.:

$$mc_i = \log \left( 1 - s_i \right) + \gamma n_i - \log (1 - \alpha) - \log (1 - \tau_i) + \log \left( Y_i^{\text{in}} - G_i \right) - a_i$$  (60)

If we express the previous expression in log-deviation from the steady state values, we obtain:

$$0 = \left( \frac{1}{1 - \kappa_i} + \gamma \right) Y_i^{\text{in}} - \frac{\kappa_i}{1 - \kappa_i} G_i - (1 + \gamma) a_i$$  (61)
Finally, the closed solution for the fully flexible prices output for country $i$ and for the currency union are given by:

\[
\hat{\bar{g}}^i = \left( \frac{\hat{\bar{g}}^i + (1 + \gamma) \hat{a}^i}{1 - \gamma} \right)
\]

\[
\hat{\bar{g}}^u = \left( \frac{\hat{\bar{g}}^u + (1 + \gamma) \hat{a}^u}{1 - \gamma} \right)
\]

Subtracting (61) from (55) obtains

\[
\hat{mc}_i = \left( \frac{1}{1 - \kappa_i} + \gamma \right) \hat{\bar{g}}^i
\]

and similarly for the currency union:

\[
\hat{mc}_i = \left( \frac{1}{1 - \kappa_i} + \gamma \right) \hat{\bar{g}}^u
\]

where $\hat{\bar{g}}^i = y_i - y_i^\alpha$ and $\hat{\bar{g}}^u = y_i^u - y_i^\alpha$ are respectively the definitions of domestic output gap and union-wide output gap in log-deviations from the steady state values.

The previous relationships state that the log-deviation of real marginal cost from the steady state is proportional to the log-deviation of output gap from its steady state level.
3.3 Calibration

Before describing the equilibrium behavior of the prototype member economy under the framework illustrated above, we need to give a numerical value to the parameters. To this purpose we distinguish two kinds of parameters: general parameters \((m, \beta, \gamma, \sigma, \alpha, \rho, \phi, \bar{r}, \theta)\) and fiscal policy parameters \((\kappa^*, \zeta^*, \tau^*_0, \tau^*_k)\). The model is parametrized on a quarterly basis.

The subjective discount factor \(\beta\) is set to 0.99 and implies a steady state value for a long-run real annualized interest rate \(\bar{r}\) of 4\%. The elasticity of substitution among the variety of goods produced in each country \(\sigma\) is set to 6, the probability for firms of having to keep their price fixed for the current quarter was set at 0.75, implying that prices are revised on average once a year, the Frisch elasticity of labor supply \(\frac{1}{\eta}\) is set to 0.3, thus implying a steady state markup of 1.2, the average share of public consumption \(\alpha\) for EMU countries is set to 0.25. This set of parameters is consistent with the values calibrated by Gall and Monacelli (2008).

The index of openness \(m\) among EMU countries is set to 0.4, following the EMU data on the average intra-Euro area share of trade (source: Eurostat statistical data (sample 1995-2005)).
Moreover, following the real business cycle literature (King and Rebelo (1999)), we suppose a high value for the persistence coefficient of labor productivity ρ. Monetary policy parameters (ϕ_r, ϕ_y) are consistent with the empirical literature about Taylor rule in the EMU (Smets and Wouters (2003)) and are in line with the Taylor principle (ϕ_r > 1). The following table summarizes the benchmark parametrization for general parameters:

Table 1: Calibration for general parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>0.4</td>
</tr>
<tr>
<td>β</td>
<td>0.99</td>
</tr>
<tr>
<td>γ</td>
<td>3.0</td>
</tr>
<tr>
<td>σ</td>
<td>6.0</td>
</tr>
<tr>
<td>α</td>
<td>0.25</td>
</tr>
<tr>
<td>ρ</td>
<td>0.95</td>
</tr>
<tr>
<td>φ_r</td>
<td>1.7</td>
</tr>
<tr>
<td>φ_y</td>
<td>0.125</td>
</tr>
<tr>
<td>θ</td>
<td>0.75</td>
</tr>
<tr>
<td>r*</td>
<td>0.04</td>
</tr>
</tbody>
</table>

In the next table we calibrate fiscal policy parameters:

Table 2: Calibration for fiscal policy parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ_n</td>
<td>0.35</td>
</tr>
<tr>
<td>τ_k</td>
<td>0.24</td>
</tr>
<tr>
<td>ζ</td>
<td>0.80</td>
</tr>
<tr>
<td>ξ</td>
<td>0.49</td>
</tr>
<tr>
<td>κ</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Tax rate on labor τ_n is set equal to the average annual implicit tax rate on labor employed (0.35) for the Euro area, tax rate on dividends and bond yields τ_k is set equal to the average implicit tax rate on capital income (0.24) for the Euro area.

The persistence coefficient in public expenditure ζ is set to 0.80, whereas we let the feedback on debt parameter κ assume the value 0.01 and the counter-cyclical parameter ξ assume the value 0.49: all these values are consistent with the estimates of Gali and Perotti (2003) for the sample 1992-2001 in EMU countries.

In the next section we discuss the dynamic behavior of the model in the presence of a technology shock under the policy rules described above.
In this section we discuss the dynamic equilibrium behavior of a representative member economy under the model discussed above, by resorting to a series of dynamic simulations with the parameterization described in the previous section. In particular, we focus our attention on the responses of macro and fiscal policy variables (public expenditure, tax revenues, public debt and the ratio public debt/GDP) to a domestic technology shock.

The set of figures shown in Appendix (Section 7.3.) displays the dynamic response of output, output gap, real public debt, the ratio debt/GDP, total private consumption, real public expenditure, hours worked, real wages, real marginal costs, real tax revenues and real profits in the aftermath of a technology shock.

The rigidity in aggregate demand resulting from the stickiness of the domestic price level leads technology shock (that in this framework is a labor productivity shock) to generate a negative comovement between hours worked and productivity. This result is also supported by strong empirical evidence (Gali (1999) and Francis and Ramey (2005) among others). The intuition for this result is straightforward. When a technology shock hits the economy, the increase in productivity determines a fall in real marginal costs, domestic prices decrease, as a consequence of a right-shift in the aggregate supply curve and following the decrease in real marginal costs. Aggregate demand and private consumption increase, due to the fall in domestic prices. Nevertheless, aggregate demand and prices, given the nominal rigidities, for which only a fraction $1 - \theta$ of firms reset their prices, change less than they would under fully flexible prices. Aggregate output increases, but by less than in the absence of price rigidities: for this reason, a contraction in the output gap occurs. Furthermore, because labor is more productive with a consequent increase in the real wages, the firms will require less labor input. As a consequence, output does not increase in the same proportion of the productivity shock. On the other hand, real profits increase due both to the positive shift of output and to the contraction in real marginal costs. Real tax revenues are procyclical. On the contrary, public debt is countercyclical, that is, every time a positive productivity shock hits the economy and shifts upwards domestic output, government debt shows a negative deviation from its steady state value: an increase in output, leading to an increase in real tax revenues, with a countercyclical public consumption, generates a countercyclical movement of the stock of public debt. In this way, the ratio debt/GDP decreases for some periods, due to the contemporaneous contraction of the real stock of public debt and the increase in output, and then reverts to the original steady state value. Thus, public expenditure, together with taxation, plays the role of a "smoother" for output cyclical fluctuations.
5 A WELFARE ANALYSIS

This section aims to evaluate the performances of our fiscal and monetary rules, by relying on a welfare-based criterion referred to country $i$ and using a second-order approximation to the utility losses of the consumers. In order to measure these utility losses, we make use of a welfare function defined here as the discounted sum of the utilities across households:

$$F = \sum_{t=0}^{\infty} \beta^t U(C_i^t, N_i^t, G_i^t)$$  \hspace{1cm} (64)

The benchmark values against which we measure the welfare losses associated to our policy rules are referred to an economic framework without distortionary taxation, with zero stock of public debt and with lump-sum taxation able to finance public consumption at its optimal level. Prices are fully flexible and, as argued by Galli and Monacelli (2008), in this framework monetary policy is completely neutral.

Furthermore, firms’ market power given by the monopolistic competition is completely offset by the employment subsidy set at its optimal value, as will be shown below.

Galli and Monacelli (2008) demonstrate that in this context the optimal value for each variable is derived by the solution of the social planner’s problem, that, in turn, is supported by a decentralized fully flexible price equilibrium.

In fact, the social planner’s optimal allocation for the currency union is given by the solution of the following maximization problem:

$$\max \int_0^1 U(C_i^t, N_i^t, G_i^t) \, dt$$

s.t. $Y_i^t = A_i N_i^t$

s.t. $Y_i^t = C_i^t + \int_0^1 C_i^{t, \epsilon} \, d\epsilon + G_i^t$

for all $\epsilon[0, 1]$
The solution obtained is given by the following set of relationships for all $i, f \in [0, 1]$:

\[ N^i = 1 \]  \hspace{1cm} (65)  
\[ Y^i = A^i \]  \hspace{1cm} (66)  
\[ C^i_{t,t} = (1 - \alpha) (1 - m) A^i_t \]  \hspace{1cm} (67)  
\[ C^i_{t,t+1} = (1 - \alpha) m A^i_t \]  \hspace{1cm} (68)  
\[ G^i_t = \alpha A^i_t \]  \hspace{1cm} (69)  
\[ C^i_t = (1 - \alpha) (A^i_t)^{1-m} (A^i_t)^m \]  \hspace{1cm} (70) 

The aggregation of the previous results over countries leads to the union-wide optimal allocation:
\[ Y_i^* = A_i^* \]  \hspace{1cm} (71)
\[ C_i^* = (1 - \alpha) (A_i^*) \]  \hspace{1cm} (72)
\[ G_i^* = \alpha A_i^* \]  \hspace{1cm} (73)

Moreover, the efficient allocations of the terms of trade is given by:
\[ \left( \frac{P_{i,1}^*}{P_{i}^*} \right)^m = \left( \frac{C_i^*}{C_i^*} \right)^{\frac{-\sigma}{\sigma - 1}} = \frac{A_i^*}{A_i^*} \]  \hspace{1cm} (74)

The corresponding optimal decentralized allocation reads as:
\[ \frac{\sigma - 1}{\sigma} = MC_i \]
\[ \frac{\sigma - 1}{\sigma} = \frac{(1 - s^i)}{A_i^*(1 - \alpha)} C_i^* (N_i^*)^{\gamma} \left( \frac{P_{i,1}^*}{P_i^*} \right)^m \]
\[ \frac{\sigma - 1}{\sigma} = \frac{(1 - s^i)}{A_i^*(1 - \alpha)} C_i^* (N_i^*)^{\gamma} Y_i^* - \frac{G_i^*}{C_i^*} \]
\[ \frac{\sigma - 1}{\sigma} = \frac{(1 - s^i)}{(1 - \alpha)} \left( 1 - \left( \frac{G_i^*}{Y_i^*} \right) \right) (N_i^*)^{1+\gamma} \]  \hspace{1cm} (75)

If the employment subsidy is set at a level
\[ s^i = \frac{1}{\sigma} \]  \hspace{1cm} (76)
and government spending follows the following rule,
\[ G_i^* = \alpha Y_i^* \]  \hspace{1cm} (77)

i.e. the fraction of output used for public consumption is exactly equal to the weight given to it in the utility function of the households, then the fully flexible price equilibrium in each country supports as an equilibrium the centralized allocation. In our model the presence of further distortions besides the monopolistic competition, such as the different kinds of distortionary taxation, would lead to steady state equilibrium values for each variable not efficient both from the viewpoint of the social planner and the decentralized economy. Hence, to make a comparison between our model with fiscal and monetary rules, price stickiness and market power distortions and a scenario with fully flexible prices, balanced budget public accounts, i.e. with lump-sum taxation able to finance public expenditure at a level indicated by (77), it is important to impose, for each variable involved in the model, the efficient steady state value\(^9\), that is the value corresponding to the solution of the social planner’s problem\(^10\).

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\(^9\) We thank Pierpaolo Benigno for his important suggestions on this part of the paper.
\(^10\) These values correspond to the ones expressed in the relationships (77-74) without the timing subscript.
As shown in Gali and Monacelli (2008) a second order approximation to (64) can be rewritten as the average utility losses of union households resulting from fluctuations about the efficient steady state in the following functional form:\footnote{11}{For the analytical derivation of the welfare function, see the Appendix in Gali and Monacelli (2008), Gali (2008) and Woodford (2003).}

\[ \Theta \simeq \frac{1}{2} \sum_{i=0}^{\infty} \beta^i \left( \frac{\sigma}{\alpha} \left( \pi_i^* \right)^2 + (1 + \gamma) \left( \bar{y}_i^* \right)^2 + \frac{\alpha}{1 - \alpha} \left( \bar{f}_i^* \right)^2 \right) + \text{tips} \]  

(78)

where \text{tips} denotes terms that are independent of policy and \( \bar{f}_i^* = (g_i^* - y_i^*) - \log \alpha \) is defined as the fiscal gap. This variable represents the share of output used for public consumption less the amount of public expenditure to which the households give a weight in the utility function (\( \log \alpha \)) and for this reason it represents an inefficient gap.

Gali and Monacelli (2008) show that, in the presence of sticky prices, the combined monetary-fiscal policy mix able to maximize the average welfare of union households must lead at the union level to a constant (zero) value for the output gap, inflation and fiscal gap. Anyway, the same authors argue that the union-wide equilibrium in general cannot be an equilibrium under the optimal policy for each member country: in this case the second-best allocation of the output gap and fiscal gap will have a non trivial equilibrium dynamics as for the union level. This equilibrium dynamics can be described through a series of dynamics simulations, given an appropriate calibration for the model parameters.

Taking the expected value on both side of (78) at time 0, we obtain the average welfare loss per period given by the following linear combination of domestic inflation, output gap and fiscal gap variances:

\[ L = \frac{1}{2} \left[ \frac{\sigma}{\alpha} \text{var} (\pi_i^*) + (1 + \gamma) \text{var} (\bar{y}_i^*) + \frac{\alpha}{1 - \alpha} \text{var} (\bar{f}_i^*) \right] \]  

(79)

Using (79), given the monetary and fiscal rules together with a calibration for the model’s parameters like the one described above, it is possible to compute the second order moments of the simulated time series for output gap, inflation and fiscal gap, in order to derive the corresponding welfare losses associated to these rules\footnote{12}{The artificial data generated by the simulation process are also filtered with the Hodrick-Prescott technique.}.

Table 3 reports the measures of domestic inflation, output gap and fiscal gap variances, weighted by specific coefficients, together with the percentage contributions to welfare losses in round brackets: in column "A" we analyse the effects of the presence of a Taylor rule (23) in isolation with lump-sum taxation able to finance public consumption and zero stock of public debt. In column "B" we show the effects of fiscal rules (42 and 43) in isolation with a monetary policy that simply pursues an "interest rate peg" to its long-run equilibrium value, whereas column "C" evaluates the joint effects of the monetary and fiscal rules.
Table 3 : Contributions to Welfare Losses

<table>
<thead>
<tr>
<th></th>
<th>Taylor Rule (A)</th>
<th>Fiscal Rules (B)</th>
<th>Taylor Rule + Fiscal Rules (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2} \text{var} (\pi)$</td>
<td>0.18 (28.57%)</td>
<td>0.87 (91.58%)</td>
<td>0.01 (25%)</td>
</tr>
<tr>
<td>$\frac{1}{2} (1 + \gamma) \text{var} (\bar{y})$</td>
<td>0.20 (31.75%)</td>
<td>0.05 (5.26%)</td>
<td>0.02 (50%)</td>
</tr>
<tr>
<td>$\frac{1}{2} \text{var} (\bar{f})$</td>
<td>0.25 (39.68%)</td>
<td>0.03 (3.16%)</td>
<td>0.01 (25%)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>0.63</td>
<td>0.95</td>
<td>0.04</td>
</tr>
</tbody>
</table>

From a first inspection of the table, it is evident how the mix of fiscal and monetary rules (Column C) reduces welfare losses by more than under the single rules in isolation and so it should be preferred. The comparison between fiscal rules and monetary rule shows that fiscal rules generate larger welfare losses than Taylor rule. Furthermore from an analysis of Column A and B it emerges that the Taylor rule is able to better reduce the fluctuations in domestic inflation in comparison with fiscal rules, whereas both output gap and fiscal gap show smaller variations in the presence of fiscal rules than under Taylor rule. Finally, under the monetary rule, fiscal gap has the most important role in explaining welfare losses whereas under fiscal rules, this role belongs to domestic inflation.

From this picture two key results emerge: 1) the combination of a monetary policy, that positively responds to inflation and output gap, and fiscal rules made up of distorting taxation on labor, dividends and interests on public bonds and of a countercyclical and debt-stabilizing public consumption is welfare improving with respect to the same fiscal and monetary rules considered in isolation; 2) in the presence of our monetary rule, domestic inflation variance falls by more than in the presence of fiscal rules, whereas output and fiscal gap fluctuations are better smoothed by our fiscal rules on public expenditure and taxation. Therefore, in a currency union scenario like the EMU, the common monetary policy should mainly focus on inflation stabilization, whereas fiscal policy, institutionally decentralized at a country level, should focus on output gap stabilization. The latter objective can indeed be reached even in the presence of fiscal rules focused not only on business cycle stabilization but also on debt stabilization, consistently with the SGP.

These findings have some similarities with those of Ferrero (2009) and Gali and Monacelli (2008) among others. Ferrero (2009) finds that, in the presence of a monetary rule that positively reacts to inflation and output gap, with sticky prices and with a fiscal constraint on real debt such that it positively reacts to output gap, such fiscal policy leads to welfare gains if compared to balanced budget rules, whereas monetary policy better smooths inflation and hence should focus on price stability.

Gali and Monacelli (2008) argue, from a welfare analysis, that in a currency union like the EMU and in the presence of sticky prices optimal policy requires a common monetary policy with the aim to stabilize inflation at a union-wide level and a fiscal policy oriented to output gap stabilization, which is desirable both from the viewpoint of each country member and from the union as a whole.
6 CONCLUSIONS

This paper develops a New-Keynesian multicountry model applied to the EMU context with sticky prices and the presence of rules related to fiscal and monetary policy. The former is managed by the government sector, institutionally decentralized at a single country level, that makes use of distortionary taxation on labor, on dividends and interests on public bonds and of public consumption following a countercyclical and debt-stabilizing behavior. The latter is under the control of the common monetary authority, that follows a standard Taylor rule.

The first key result is that under a favorable labor productivity shock able to push up output, the real stock of public debt with respect to GDP decreases, i.e. it is countercyclical and hence stabilizing.

From a welfare analysis of the policy rules, we evaluate the welfare contribution, in terms of welfare losses, of: a) a standard Taylor rule in a scenario without public debt and with lump-sum taxation able to finance public consumption, b) a countercyclical and debt-stabilizing fiscal rules with an interest rate pegging policy of the common central bank and c) the combination of fiscal and monetary rules.

The results obtained show that i) in the presence of our monetary rule alone, domestic inflation fluctuations are better smoothed than in the presence of our fiscal rules; ii) output gap variance is smaller in the presence of fiscal rules alone than whenever only the monetary rule is present; iii) the fiscal-monetary policy mix made up of our rules is able to lower welfare losses by more than what the monetary and fiscal rules would do in isolation.

The policy implications of these results are that i) in a currency union, like the EMU, monetary policy should have the objective of inflation stabilization, as institutionally indicated in the Maastricht Treaty; ii) fiscal policy should focus on output gap stabilization. Importantly, this aim can be pursued in the presence of fiscal rules not univocally oriented to business cycle stabilization (one of the purposes of fiscal policy according to the theory of public finance of Musgrave (1959)), but also to debt stabilization, as prescribed by the SGP.

The theoretical structure described above calls for further analysis on several points.

The model ignores capital accumulation and stickiness is only confined to prices and not to wages.

It could be useful to make a distinction in the government expenditure rule between current public expenditure and capital public expenditure, in order to define different behaviors of these components.

Another important extension deals with a richer source of economic fluctuations. In particular, I plan to allow for non-technology shocks and see how these may be informative.

Furthermore, the restrictive assumption of non tradeable bonds and ownership rights of the firms could be relaxed in favor of an hypothesis of almost complete markets of financial assets.

Moreover, by moving from a positive analysis to a normative one, the loss
function (79) could be used to find the coefficients of the policy rules able to minimize the welfare losses.

We plan to further examine some of these issues in our future research.
REFERENCES


7 APPENDIX

7.1 Profit Maximization problem in steady state

\[
\begin{align*}
\max_{Y^i(j)} & \quad \left[ (\bar{Y}^i(j))^{\frac{1}{\sigma}} \times \bar{P}^i \right] \\
\frac{\partial \bar{Y}^i(j)}{\partial Y^i(j)} & = 0 \quad : \left( \frac{\sigma - 1}{\sigma} \right) (\bar{Y}^i(j))^{-\frac{1}{\sigma}} \times \bar{P}^i \times \frac{\bar{W}^i}{\bar{A}^i} = 0 \\
\left( \frac{\sigma - 1}{\sigma} \right) (\bar{Y}^i(j))^{-\frac{1}{\sigma}} \times \bar{P}^i \times (\bar{Y}^i)^{-\frac{1}{\sigma}} & = \frac{\bar{W}^i}{\bar{A}^i} \\
\left( \frac{\bar{Y}^i(j)}{\bar{Y}^i} \right)^{-\frac{1}{\sigma}} \times \bar{P}^i \times \left( \frac{\sigma - 1}{\sigma} \right) & = \frac{\bar{W}^i}{\bar{A}^i} \\
\bar{P}^i(j) \times \left( \frac{\sigma - 1}{\sigma} \right) & = \left( \frac{\bar{W}^i}{\bar{A}^i} \right) \\
\bar{P}^i(j) & = \left( \frac{\bar{W}^i}{\bar{A}^i} \right) \left( \frac{\sigma}{\sigma - 1} \right) \\
\left( \int_0^1 \bar{P}^i(j)^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}} & = \left( \int_0^1 \left( \frac{\bar{W}^i}{\bar{A}^i} \right) \frac{\sigma}{\sigma - 1} \, dj \right)^{\frac{1}{1-\sigma}} \\
\bar{P}^i & = \left( \frac{\bar{W}^i}{\bar{A}^i} \right) \left( \frac{\sigma}{\sigma - 1} \right) \quad (80) \\
\frac{\bar{W}^i}{\bar{A}^i} & = \frac{\sigma - 1}{\sigma} = MC^i \quad (81)
\end{align*}
\]

The expression (80) states that, in steady state, the level of price \( \bar{P}^i \) given by the product of marginal costs \( \left( \frac{\bar{W}^i}{\bar{A}^i} \right) \) and markup \( \left( \frac{\sigma}{\sigma - 1} \right) \).
7.2 Taylor expansion of $\log (Y^i_t - G^i_t)$

Let $\kappa = \frac{G}{Y}$ the steady state government spending share. Define $\hat{g}^i_t = \log \frac{Y^i_t}{Y}$ and $\hat{G}^i_t = \log \frac{G^i_t}{G}$. A first-order Taylor expansion of $\log (Y^i_t - G^i_t)$ about the steady state yields:

$$\log (Y^i_t - G^i_t) = \log \left((1 - \kappa_t) \bar{Y}\right) + \frac{1}{1 - \kappa_t} \left(\frac{Y^i_t - \bar{Y}}{\bar{Y}}\right) - \frac{\kappa_t}{1 - \kappa_t} \left(\frac{G^i_t - \bar{G}}{\bar{G}}\right)$$

$$\log (Y^i_t - G^i_t) = \log \left((1 - \kappa_t) \bar{Y}\right) + \frac{1}{1 - \kappa_t} \left(\hat{g}^i_t - \kappa_t \hat{G}^i_t\right)$$

7.3 Impulse response functions to a shock in technology

![Impulse response functions to a shock in technology](image)
Impulse responses to a shock in technology

Percent deviation from steady state

Years after shock

Impulse responses to a shock in technology

Percent deviation from steady state

Years after shock
Ministry of Economy and Finance

Department of the Treasury

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