Firms’ Finance, Cyclical Sensitivity, and the Role of
Monetary Policy

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Abstract

This paper analyzes new considerations for monetary policy in a model with firms’
heterogeneity in financing operation: some firms are cash constrained and need to
borrow in order to operate, although others are not. In such a model, aggregate pro-
ductivity shocks and monetary policy shocks affect differently the firms’ borrowing and
production decisions, depending on the way firms finance their inputs. We find that
although the cash unconstrained firms respond procyclically to productivity shocks, the
cash constrained firms may reduce labor demand during an expansion. When a positive
productivity shock increases the wage, it creates a burden to the cash constrained firms
which need to borrow in order to finance the higher wage. A monetary policy shock
has very different effects; the cash constrained firms react procyclically and severely.
This is because monetary policy affects the loanable funds market which is crucial for
the behavior of the cash constrained firms. Given that smaller firms are usually more
cash constrained, the model can account for recent empirical findings (Moscarini and
Postel-Vinay, 2012) that larger firms’ employment is more cyclical than smaller firms’
employment, and that small firms increase employment during recessions. At the same
time this paper accounts for earlier findings that smaller firms react more severely than
the larger ones after a monetary policy shock (Gertler and Gilchrist, 1994). Given these
reactions, optimal monetary policy in our model responds to the productivity shocks in
order to minimize the external finance premium and correct the inefficiency the limited
participation creates.

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1 Introduction

How does the economy react to productivity shocks, and then how, if at all, should monetary policy respond to these shocks? We attempt to reexamine these macroeconomic questions using a simple limited participation model with heterogeneous firms, where firms differ in the way they finance their production; some use internal finance but others have to use external finance. We find that optimal, welfare maximizing monetary policy reacts to economic conditions in order to adjust the funds in the loanable funds market and balance the external finance premium, correcting in this way the inefficiency created because of the limited financial market participation.

A large percentage of firms depends on borrowing in order to finance operations. For example, Rajan and Zingales (1998) reports that more than 30% of the operation of the US firms was depending on external finance during the 80’s. Work from Gertler and Gilchrist (1994) and Bernanke et al. (1996) shows how monetary policy affects an economy where firms use costly external finance. Specifically, they conclude that in a downturn, identified using Romer and Romer (1989) monetary policy shocks dates, smaller firms react more than larger firms by more severely reducing their economic activity. However, a more recent study by Moscarini and Postel-Vinay (2012) finds that large firms react more sharply compared to smaller firms, to the business cycle. That work identifies recessions using the NBER dates, and argues that large firms are more cyclically sensitive than small firms; they create more jobs during an expansion although they lay off extensively during a recession. In addition, Moscarini and Postel-Vinay (2012) finds that small firms create jobs during a recession, justifying the widespread argument that small businesses fuel jobs’ creation during recessions. Chari et al. (2007) studies large and small firms’ behavior during the cycle and after monetary policy shocks. The study finds that larger firms react more than smaller ones to the cycle as identified by NBER dates, but smaller firms react more than larger ones to monetary policy shocks. This paper is motivated by these observations and presents a model in order to address the recent empirical evidence.

Previous literature on financial frictions and monetary policy has focused on the costly state verification of firms affected by idiosyncratic shocks (Carlstrom et al., 2010, Fiore et al., 2011, Bernanke et al., 1999). Our work concentrates on heterogenous firms in terms
of their ability to use internal finance. This type of heterogeneity affects the way that aggregate productivity and monetary policy shocks transmit across the two types of firms.

Our starting point is the idea that firms need to borrow funds in order to pay their input suppliers; that reveals a friction related to the usefulness of money: information frictions do not allow firms to pay the input suppliers with credit. That is, firms need to borrow cash in order to pay for the labor they hire because the suppliers would not agree to offer their inputs for the firms’ IOU. Using this observation in a form of a cash-in-advance constraint in production Fuerst (1992) shows that monetary policy can have real effects.

In addition, Williamson (2005), in a modified cash-in-advance model where the monetary authority affects only some households, studies the asymmetric effects that monetary policy has across agents in the economy. Similar ideas are studied in Williamson (2006), using a monetary search model and deriving the cash-in-advance constraints endogenously. Our model studies asymmetric effects of monetary policy too. It is inspired by the large family models (Lucas, 1990) and the limited participation literature (Fuerst, 1992; Christiano and Eichenbaum, 1995), where firms are subject to a cash-in-advance constraint and need to borrow money in order to pay upfront the suppliers of inputs. Our work also derives intuition from Palivos et al. (1993), who use a modified cash-in-advance model, where only a fraction of the firms’ operation is financed with credit, in order to study velocity changes. In our model firms differ in their dependence on borrowing. It is a limited participation model where only the cash constrained firms use the loanable funds market, and thus only the cash constrained firms are directly affected by monetary policy.¹

Concerning this economy’s reaction to productivity shocks, the model predicts that both types of firms increase their production in response to an expansionary productivity shock. However, employment in the cash constrained firms decreases, although it increases in the cash unconstrained firms. Total employment increases and working hours flow from the cash constrained to the unconstrained firms. On the other hand, during a recession production of both types decreases and total employment decreases as well. Then, working hours flow from the cash unconstrained to the cash constrained firms. If we use small firms as a proxy for the cash constrained firms and large firms as a proxy for the cash

¹We can think that cash unconstrained firms are indifferent between using credit or using the loanable funds market for borrowing funds with zero interest.
unconstrained firms then the model’s results match Moscarini and Postel-Vinay (2012) findings.

The mechanism behind the results is as follows: Cash unconstrained firms’ sharp reaction to productivity shocks is accompanied by a sharp change in the wage. For example, during expansion cash unconstrained firms increase their labor demand increasing the competitive wage. Higher wage burdens the operation of the cash constrained firms which have to borrow in order to finance their labor input. Cash constrained firms decrease their labor demand and their demand for loanable funds, letting the nominal interest decline. On the other hand, after a negative productivity shock cash unconstrained firms reduce their labor demand and put downward pressure to the wage. This benefits the cash constrained firms which now have to finance lower wage and thus increase their labor demand. Using again the same proxies as before, our results match Moscarini and Postel-Vinay (2012) empirical findings that in a recession, as measured by NBER dates, it is the small firms that increase employment.

The fact that our model’s predictions match empirical evidence showing that large firms are more cyclical than small firms does not mean that the model contradicts Gertler and Gilchrist (1994)’s finding. As Chari et al. (2007) has indicated, not all business cycles are created by monetary policy shocks. Our model takes into account this distinction into account and shows that a monetary policy shock has very different effects than the productivity shock. By changing the money supply monetary policy affects directly the cash constrained firms because it changes the amount of funds available for them to borrow. A monetary expansion increases the loanable funds’ supply, reducing the nominal interest rate and creating the liquidity effect. Cash constrained firms increase employment and production.

At first glance our results might seem incompatible with evidence concerning the last recession (December 2007-June 2009). Fort et al. (2012) finds that this recession had been different from the previous ones, having small firms hit very roughly in their employment. This last recession, much more connected with the financial markets, has created a credit crunch decreasing the amount of credit available for cash constrained firms to borrow. The shock does not seem as a productivity shock in the way the model addresses productivity.
shocks. On the contrary, it looks like a shock in the available credit, a shock similar to the one tight monetary policy can create. Having this in mind our model is not incompatible, but can reconcile also the evidence of the recent recession.

This work suggests that optimal monetary policy reacts to productivity shocks. This is because the productivity shocks affect differently the two types of firms, although it is efficient to behave identically. Optimal monetary policy expands offering more loanable funds after a positive productivity shock; it does that in order to benefit the cash constrained firms who need to finance higher wage. On the other hand, optimal monetary policy tightens after a negative productivity shock; it does that in order to prevent outflow of labor from the cash unconstrained to the cash constrained firms, which now have to finance lower wage.

The rest of the paper is organized as follows. Section 2 introduces the model economy. Section 3 studies the effects that a productivity shock has for the two types of firms. Section 4 explores the effects that a money supply change has. Section 5 studies the optimal monetary policy. Section 6 concludes.

2 The Model Economy

We employ a representation of the economy inspired by Lucas (1990), in the sense that the model economy is populated by infinitely lived large families. All families are identical and seek to maximize their lifetime utility:

$$E_{0}\{\sum_{t=0}^{\infty} \beta^{t}[U(c_1^t) + V(c_2^t) - D(H_t)]\},$$

where the expectation refers to the economy’s shocks, which we are going to specify later, and $0 < \beta < 1$ is the family’s discount factor. As we are going to describe in more detail below, there are two industries in this model, producing two consumption goods. $U(.)$ denotes utility derived from consuming $c_1^t \geq 0$, the good that industry 1 produces at time $t$, with $U'(.) > 0$, $U''(.) < 0$, $U'(0) = \infty$, $U'(\infty) = 0$. Similarly, $V(.)$ denotes utility derived from consuming $c_2^t \geq 0$, the good that industry 2 produces at time $t$, with $V'(.) > 0$, $V''(.) < 0$, $V'(0) = \infty$, $V'(\infty) = 0$. Finally, $-D(.)$ denotes disutility from
There are two industries producing consumption goods in this model: the cash unconstrained industry and the cash constrained industry. Firms in either industry maximize profits using the production functions \( f^c(h^c_t, \theta_t) = \theta_t h^c_t \) and \( f^u(h^u_t, \theta_t) = \theta_t h^u_t \) where \( h^c_t \geq 0 \), \( h^u_t \geq 0 \) is the labor hired by each cash constrained and each cash unconstrained firm respectively; \( 0 < \theta_t < \infty \) for every \( t \), denotes the productivity shock hitting the economy and affecting the production process of firms in both industries. There are \( \lambda \) firms operating in the cash constrained industry and \( \kappa \) firms operating in the cash unconstrained industry, with \( \kappa, \lambda \in [0, 1] \) and \( \kappa + \lambda = 1 \). The total labor time hired by the cash constrained and cash unconstrained industries is \( H^c = \lambda h^c_t \) and \( H^u = \kappa h^u_t \), respectively.

We need now to describe how trade takes place in this model economy. Each household consists of three members: a shopper, a worker and a financial transactor. All family members leave the family at the same time, in order for each of them to execute a specific task. The shopper receives part of the family’s initial money balances and travels to the goods markets in order to buy consumption goods from the two industries. The financial transactor receives a fraction of the initial money balances in order to make loans in the loanable funds market. Finally, the worker leaves the family in order to work for the two types of firms.

The firms hire labor for the competitive wage, produce, and ship their products to the goods markets. The cash constrained firms borrow cash from the loanable funds market in order to produce, which they return at the end of the period paying the market’s interest rate.

In order for money to be valuable, we assume a cash-in-advance constraint in consumption, and part of the production. Specifically, there is a cash-in-advance constraint in consumption, so that the family cannot buy consumption goods in period \( t \) with money earned in period \( t \); instead, cash balances need to be available in advance. Moreover, there is a cash-in-advance constraint in the production of the cash constrained firms. These firms, contrary to the cash unconstrained firms, cannot buy labor input with credit, but need to have cash balances in advance in order to pay the wage bill to the labor they employ.

The cash constrained firms may borrow cash balances from the loanable funds market.
The quantity of loanable funds supplied comes from money balances that the financial transactor carries, and transfers that the monetary authority injects. Specifically, the monetary authority changes the money supply from time \( t \) to time \( t + 1 \) with a rate equal to \( 0 \leq \mu_t < \infty \). Money growth is injected in the loanable funds market affecting the amount of funds available for the constrained firms to borrow. In this way, monetary policy affects directly only the cash constrained firms, which are the ones that participate in the financial markets.

The timing of the events is similar to Fuerst (1992) and Williamson (2005): The household enters period \( t \) with money balances \( M_t \), and determines the fraction of its money balances to be allocated to consumption spending, \( X_t \), and the fraction allocated for loans, \( N_t \geq 0 \). That is:

\[
M_t \geq X_t + N_t. \tag{2}
\]

Note that both these decisions are taken before the productivity and the monetary shock are realized. After deciding the money balances directed to the goods markets and to the loanable funds market, the members of the family separate. At this point, the productivity shock \( \theta_t \) and the monetary shock \( \mu_t \) are realized and uncertainty is resolved.

The shopper travels to the goods market where she uses at most \( X_t \) amount of cash for buying consumption good \( c_t^1 \) from the cash constrained firms at the price \( P_t^1 \geq 0 \), and consumption good \( c_t^2 \) from the cash unconstrained firms at the price \( P_t^2 \geq 0 \). Consumption can only be financed with cash, according to the following cash-in-advance constraint:

\[
X_t \geq P_t^1 c_t^1 + P_t^2 c_t^2. \tag{3}
\]

The financial transactor travels to the loanable funds market holding \( N_t \) amount of cash. However, the amount of loanable funds supplied, \( B_t^s \), is determined both by the financial transactor’s deposits and the monetary authority’s actions, realized as a monetary transfer of \( T_t \) units of cash, injected in the loanable funds market. Then, the total quantity of loanable funds supplied is given as follows:

\[
N_t + T_t \geq B_t^s. \tag{4}
\]
Without loss of generality we will assume that for non-negative nominal interest rate paid on loans, $R_t \geq 0$, the intermediary lends out all its cash. The profit that the intermediary attains, $N_t + T_t - B_t^s + B_t^u(1 + R_t) - N_t(1 + R_t)$, is transferred to the household at the end of the period.

The worker travels to the labor market, where she may work for a cash constrained or unconstrained firm, earning in either case the competitive wage, $W_t \geq 0$.

The family’s budget constraint is as follows:

$$M_t + W_t H_t + (1 + R_t) B_t^s + T_t + \Pi_t \geq M_{t+1} + P_t^1 c_t^1 + P_t^2 c_t^2 + B_t^s,$$

where $\Pi_t$ is the total profit of both types of industries.

As mentioned earlier, the monetary authority can intervene in the loanable funds market changing the amount of cash available for the constrained firms to borrow. Specifically, the monetary authority operates as follows:

$$\bar{M}_{t+1} = \bar{M}_t (1 + \mu_t),$$

where $\bar{M}_t$ denotes money supply at time $t$, with $0 \leq \bar{M}_0 < \infty$, and $\mu_t$ is the money growth from time $t$ to time $t + 1$. The money entering the economy is distributed as transfers, $T_t$, to the intermediary. That is:

$$\bar{M}_{t+1} = \bar{M}_t + T_t.$$

In order to focus later on the set of stationary equilibria, it is useful to introduce notation now and divide all nominal variables with the beginning of the period money stock $\bar{M}_t$. We denote the new variables with lower case letters. In addition, for convenience, we drop the time subscripts. Variables with a prime denote next period values:

$$p^1 \equiv \frac{P_t^1}{M_t}, p^2 \equiv \frac{P_t^2}{M_t}, n \equiv \frac{N_t}{M_t},$$

$$b^s \equiv \frac{B_t^s}{M_t}, w \equiv \frac{W_t}{M_t}, m \equiv \frac{M_t}{M_t}, m' \equiv \frac{M_{t+1}}{M_t+1}.$$
Using the new notation the family solves:

\[ v(m) = \max_{x,n} \mathbb{E}_{\theta,\mu} \left[ \max_{c_1,c_2,H,b^*,m'} \left[ U(c_1) + V(c_2) - D(H) + \beta v(m') \right] \right], \]  

under the following constraints:

\[ m \geq x + n, \]  
\[ x \geq p^1 c^1 + p^2 c^2, \]  
\[ n + \mu \geq b^*, \]  

and

\[ m + wH + (1 + R)b^* + \mu + \pi \geq m'(1 + \mu) + p^1 c^1 + p^2 c^2 + b^*, \]  

where \( \mu \equiv \mu_t \) is money growth but we drop the time subscript to retain consistency in notation. Also, the expectation operator \( \mathbb{E}_{\theta,\mu} \) is defined with respect to the information the household possesses exactly when entering period \( t \), i.e., before the economy’s shocks are realized.

Let \( \gamma, \gamma_1, \gamma_2, \delta \geq 0 \) be the multipliers for equations (7), (8), (9) and (10) respectively. The first order conditions for the household’s maximization problem are given below:

With respect to \( x \):

\[ -\gamma + \mathbb{E}_{\theta,\mu}(\gamma_1) = 0, \]  

with respect to \( n \):

\[ -\gamma + \mathbb{E}_{\theta,\mu}(\gamma_2) = 0, \]  

with respect to \( c^1 \):

\[ U'(c_1) - p^1 \gamma_1 - p^1 \delta = 0, \]  

with respect to \( c^2 \):

\[ V'(c_2) - p^2 \gamma_1 - p^2 \delta = 0, \]  

with respect to \( H \):

\[ -D'(H) + w\delta = 0, \]  

where \( \mu \equiv \mu_t \) is money growth but we drop the time subscript to retain consistency in notation. Also, the expectation operator \( \mathbb{E}_{\theta,\mu} \) is defined with respect to the information the household possesses exactly when entering period \( t \), i.e., before the economy’s shocks are realized.
with respect to $b^e$:
\[-\gamma_2 + \delta (1 + R) - \delta = 0,
\tag{16}\]
and with respect to $m'$:
\[-\delta (1 + \mu) + \beta \gamma' + \beta E_{(\theta', \mu')}(\delta') = 0.
\tag{17}\]

In addition to the conditions above, the multipliers’ constraints hold at each period and the relevant envelope conditions are used for solving the problem.

Equation (11) illustrates important timing effects: At the time that the household decides the amount of cash to be devoted to consumption expenditure, $x$, does not know but only expects what the change in the optimal value will be after relaxing the constraint given by equation (8). This is because at the time that the decision concerning $x$ is taken, the productivity and monetary shocks are not yet realized. This explains the presence of the expectation operator in the first order condition (11). The cost of increasing planned expenditure $x$ originates to the escaped liquidity, represented by the multiplier $\gamma$. Similar reasoning applies for equation (12).

We now turn to the firms’ profit maximization problem. There are two types of firms operating in this model economy: the cash unconstrained and the cash constrained firms. They use the production functions $f^u(h^u, \theta) = \theta h^u$ and $f^c(h^c, \theta) = \theta h^c$ respectively. The cash unconstrained firms can operate through credit. After realizing the productivity shock $\theta$ and monetary shock $\mu$, each of them decides how much labor $h^u$, to hire. They promise to pay the worker the competitive wage $w$, which they pay after they complete their production process and earn revenues. The profit for the cash unconstrained firms is given below:

$$
\pi^u = p_2 \theta h^u - wh^u.
\tag{18}
$$

The cash constrained firms, after realizing the productivity shock $\theta$ and monetary shock $\mu$, decide to borrow $b^d$ amount of cash from the loanable funds market in order to buy $h^c$ amount of labor. These firms need to acquire cash in advance in order to start their production process; they pay the competitive wage $w$ upfront for hiring workers. After they produce and sell their goods, the cash constrained firms repay their loans with net
nominal interest $R$. The profit for the cash constrained firms is as follows:

$$\pi^c = p_1 \theta h^c - w h^c - R b^d. \quad (19)$$

The cash constrained firms’ profit maximization problem is subject to a cash-in-advance constraint, $w h^c \leq b^d$, so that the amount of loanable funds demanded needs to cover at least, the labor cost of the cash constrained firms. The first order conditions imply that $R = \zeta$ and $p_1 \theta = w (1+\zeta)$, where $\zeta$ is the multiplier of the constrained firms’ cash-in-advance constraint.

The firms’ profit maximization conditions are summarized below. After the production process takes place and profit is determined, both types of firms pay profits to the household.

$$p^1 \theta = w (1 + R), \quad (20)$$

$$p^2 \theta = w, \quad (21)$$

which imply the following:

$$p^1 = (1 + R)p^2. \quad (22)$$

The last equation reveals that for the cash constrained firms, there is a wedge between the marginal product of labor and the competitive wage they offer. This happens because the cash constrained firms need to borrow cash from the loanable funds market in order to operate. Then, their decision over the amount of labor they hire depends on the nominal interest rate they have to pay. This wedge will create the difference between the price of the good that the cash constrained firms produce and the price of the good the cash unconstrained firms produce.

We will look at an equilibrium where prices $w$ and $p^1, p^2$ are strictly positive, and $n, H^u, H^c$ are interior and bounded between zero and one. Also, we let the multipliers $\gamma, \gamma_1, \delta$ and $\zeta$ be strictly positive and will verify this assumption after constructing the equilibrium.$^2$

The budget constraint binds as usually. For binding also the rest of the constraints it turns

$^2$We have already assumed that constraint (9) binds for $R_t \geq 0$. 

out that:

\[ m = x + n, \quad (23) \]

\[ p^1 c^1 + p^2 c^2 = x, \quad (24) \]

\[ n + \tau = b^\delta. \quad (25) \]

The last equation determines the loanable funds supply.

There are four markets in the model. The market clearing conditions are as follows:

The goods markets clear:

\[ e^1 = \theta H^c, \quad (26) \]
\[ e^2 = \theta H^u, \quad (27) \]

where \( H^c = \lambda h^c \) and \( H^u = \kappa h^u \).

The labor market clears:

\[ H^c + H^u = H. \quad (28) \]

The loanable funds market clears:

\[ wH^c = b^\delta. \quad (29) \]

The money market clears:

\[ m = m' = 1. \quad (30) \]

After substituting equations (23), (29) and (30) into expression (25) yields:

\[ 1 + \tau - x = wH^c, \quad (31) \]

which states that after deciding consumption expenditure, the remaining money balances, augmented by the monetary transfer, will be lent to the cash constrained firms and used for hiring the necessary labor in order to operate.

Equations (11), (13), (15) and (17) yield that:

\[ \beta E_{c, \mu} \left[ \frac{U'(c^1)^{d}}{p^{d1}} \right] = (1 + \mu) \frac{D'(H)}{w}, \]
and similarly, equations (11), (14), (15) and (17) yield that:

\[ \beta E_{\{\theta', \mu'\}} \left[ \frac{V'(c^2)}{p^2} \right] = (1 + \mu) \frac{D'(H)}{w}, \]

from which we can write the labor supply equation as follows:

\[ (1 + \mu) \frac{D'(H)}{w} = \beta \psi, \quad (32) \]

where, assuming that the productivity shock \( \theta \) and the monetary shock \( \mu \) are \( iid \), uncorrelated shocks, \( \psi = E_{\{\theta', \mu'\}} \left[ \frac{U'(c^1)}{p^1} \right] = E_{\{\theta', \mu'\}} \left[ \frac{V'(c^2)}{p^2} \right] \) is a constant, not affected by the economy’s shocks. Also, it is true that \( \psi > 0 \). In addition, in the case of an equilibrium with \( iid \) and uncorrelated shocks, and given that all the constraints bind, \( x \) does not depend on the state of the economy, and then all the variables of interest depend on the expectation of the shocks. We will solve the model given \( x \).

The above equations reveal that the discounted expected marginal utility that labor earnings yield, by either consuming the good that the cash constrained, or the one that the cash unconstrained firms produce, equals the marginal disutility of labor. This is because the cash-in-advance constraint (8) prevents the household from consuming its labor earnings during the period generated, but has to keep them as money holdings and consume them in the next period.

Similarly to Fuerst (1992), this model produces a liquidity effect. From equations (13)-(16), and (32), the nominal interest rate is as follows:

\[ 1 + R = \frac{\gamma^2 - \gamma^1}{M} \frac{V'(c^2)}{p^2} + \frac{V'(c^2)}{p^2}, \]

For \( \gamma^2 = \gamma^1 \), then the ex post value that cash has in financial and goods markets is the same, and there is no liquidity effect. For \( \gamma^2 > \gamma^1 \), i.e., for the value of cash in the financial markets being higher than what it is in goods market, then the interest rate is higher than it’s fundamentals’ Fisherian value. Note that ex ante, it is true that the value of cash in any of the two markets is the same, i.e., \( E_{\theta, \mu} \gamma^2 = E_{\theta, \mu} \gamma^1 \). However, that’s before the family observes the economy’s shocks. After the shocks are realized, there is a forecast
error, \( \gamma^2 - E_{\theta, \mu} \gamma^2 - (E_{\theta, \mu} \gamma^1 - \gamma^1) = \gamma^2 - \gamma^1 \), which is responsible for the liquidity effect. Monetary injections decrease the nominal interest rate because they are directed to the financial markets. The higher the value that cash has in financial markets compared to goods markets, the steeper is the need for loanable funds, and the larger is the decrease in the nominal interest rate.

Also, from equations (13) and (14) yields:

\[
\frac{U'(c_1)}{V'(c_2)} = \frac{p_1}{p_2},
\]

(33)
i.e., the ratio of the marginal utilities that the consumption goods of the two types of firms produce, equals the relative price of buying these goods.

Equations (24) and (33) solve for consumption \( c_1 \) and \( c_2 \), given the prices of the goods and the amount of money allocated to the buyer of the household. In addition, equation (32) solves for the choice of labor supply, given the wage.

Equations (22) and (33) imply:

\[
1 + R = \frac{U'(c_1)}{V'(c_2)},
\]

(34)
so the nominal interest rate depends on the marginal utilities of the two goods that the cash constrained and cash unconstrained firms produce. Specifically, when the household’s marginal utility for consuming the good that the cash constrained firms produce is high relative to the marginal utility from consuming the good that the cash unconstrained firms produce, then the relative price of the good that the cash constrained firms produce increases. The cash constrained firms would like to borrow more in order to finance their operation, as they know they will be selling their good for a higher price. For given supply of loanable funds, the nominal interest rate increases.

Also, from equations (21), (22), (24), (26) and (31), we can write the nominal interest rate as follows:

\[
1 + R = \frac{xc_1^1 - (1 + \mu - x)c_2}{(1 + \mu - x)c_1^1}.
\]

(35)
Combining now equations (34) and (35) yields:

\[(1 + \mu - x)c^1U'(c^1) = V'(c^2)(xc^1 - (1 + \mu - x)c^2).\] (36)

From equations (21), (26) and (31), the price of the good that the unconstrained firms produce is:

\[p^2 = \frac{1 + \mu - x}{c^1},\] (37)

which combined with equations (26) to (28), implies that equation (32) can be written as follows:

\[(1 + \mu)\frac{D'(\frac{c^1 + c^2}{\theta})c^1}{(1 + \mu - x)\theta} = \beta\psi.\] (38)

Equations (36) and (38) can be solved to determine the two unknowns, \(c^1\) and \(c^2\). Then, equation (37) solves for the price of the cash unconstrained good, equation (35) for the nominal interest rate, and given that, equation (22) solves for the price of the cash constrained good. Equations (20) or (21) solve for the wage, and equations (26) and (27) solve for the labor employed by the cash constrained and unconstrained industry respectively.

3 The Effects of a Productivity Shock

We now study the effects of a productivity shock in the economy described above. We find that a positive productivity shock (an increase in \(\theta\)) increases the production of both cash constrained and unconstrained firms. This is because the positive productivity shock makes both types of firms more productive. Given that firms produce inelastically, each level of employment produces more good and both types of firms tend to increase their production. There is downward pressure on the prices of the goods and thus consumers agree to buy the extra amount of goods produced. Note that the binding cash in advance constraint (24) prevents consumers from using for any other purpose the predetermined, before observing the productivity shock, amount of money \(x\) that is assigned for consumption.

Although production increases for both types of firms, the same is not true about employment. We are going to assume that \(-\frac{c^1U''(c^1)}{U'(c^1)} > -\frac{c^2V''(c^2)}{V'(c^2)}\) so that \(\frac{dw}{d\theta} > 0\) and the
productivity shock increases the equilibrium wage. Higher wage increases the household’s labor supply (see equation (32)), which could be directed to the cash unconstrained and/or the cash constrained firms. However, from equation (31) we see that higher wage translates to a decrease in the constrained firms’ employment opportunities. This is because the household has already decided, before the productivity shock is realized, the amount of cash to deposit to the intermediary. Given that the amount available to borrow is predetermined, higher wage means that the constrained firms will have to decrease the labor they employ (see (31)). This in turns puts downward pressure in the demand for loanable funds from the cash constrained firms. Lower demand for loanable funds translates in lower nominal interest rate, given the supply of loanable funds which is affected only by the monetary authority (see (25)). Note that at this point we assume that monetary policy acts independently, and the monetary policy shock is uncorrelated with the productivity shock. In section 5 we examine monetary policy’s response to the productivity shock.

The results are justified by the dissimilarity in the way the two types of firms finance their operation. The productivity shock affects the two industries differently. The cash unconstrained firms wish to increase their production after a high productivity shock is realized, driving the competitive wage in high levels. These firms hire higher amount of labor, and sell their product in a lower price.

Yet, this wage increase is bad news for the cash constrained firms which need to borrow in order to pay for the labor they hire. Labor is more expensive for the cash constrained firms, compared to the cash unconstrained ones, as the cash constrained firms need to pay also the interest rate. The productivity shock, through the increase in the competitive wage will decrease the labor hired by the cash constrained firms.

It is interesting to see also the flow of employment during the cycle. The positive productivity shock increases overall employment, with all the new hours directed to the cash unconstrained firms. In addition, there is a flow of hours from the cash constrained to the cash unconstrained firms. Relative employment ($\frac{H^C}{\overline{H}}$) decreases for the cash constrained firms, but increases for the cash unconstrained ones ($\frac{H^U}{\overline{H}}$).

The opposite results hold in a downturn. Production of both constrained and unconstrained firms decreases. The downward pressure that unconstrained firms put on the
wage is now beneficial for the constrained firms, which increase their labor input. Overall employment decreases, but cash constrained firms’ employment, and relative employment increases. During the downturn there is a flow of working hours from the cash unconstrained to the cash constrained firms.

Our results about the effect of productivity shocks on cash constrained and cash unconstrained firms are in line with the empirical evidence of Moscarini and Postel-Vinay (2012) and Chari et al. (2007), if we assume that the cash constrained firms are small firms and the cash unconstrained firms are large ones. In addition, our results provide theoretical background for Moscarini and Postel-Vinay (2012)’s finding, but also popular argument, that during recessions, small businesses are the fuel of jobs growth.

This does not mean that our results are not in line with Gertler and Gilchrist (1994). Our model makes the distinction between productivity and monetary policy shocks. Productivity shocks are found to affect more the cash unconstrained firms, similarly to Moscarini and Postel-Vinay (2012)’s finding that large firms react more than small firms to the NBER dates. However, cash constrained firms are more affected by monetary policy shocks, similarly to Gertler and Gilchrist (1994)’s finding that small firms react more to monetary policy shocks dates, as we will see in the next section.

In short, the response of the endogenous variables to the productivity shock is:

\[
\frac{dc^1}{d\theta} > 0, \quad \frac{dc^2}{d\theta} > 0, \quad \frac{dp^1}{d\theta} < 0, \quad \frac{dp^2}{d\theta} < 0, \quad \frac{d(1+R)}{d\theta} < 0, \quad \frac{dw}{d\theta} > 0, \quad \frac{dH^c}{d\theta} < 0, \quad \frac{dH^u}{d\theta} > 0, \quad \frac{dH}{d\theta} > 0.
\]

4 Monetary Policy Effects

We now examine how a change of the monetary authority’s decisions will affect the economy and the two types of firms. The cash constrained firms are directly affected by monetary policy as they use the loanable funds market through which monetary policy operates. The larger the monetary authority’s expansion, the larger the amount of funds available to the cash constrained firms to borrow. The mechanism is similar with that explored by Gertler and Gilchrist (1994), but refers only to a part of the firms’ population.

Higher money supply increases the production of the cash constrained firms, which now have more funds available to borrow and finance their production. Due to the binding cash in advance constraints (23) and (25), an increase in the transfers the financial intermediary
gets translates into an equal increase in the supply of loanable funds, which decreases the interest rate. This is the liquidity effect that as usually, limited participation models exhibit. In short: \( \frac{dc}{d\mu} > 0, \frac{d(1+R)}{d\mu} < 0, \frac{dH^c}{d\mu} > 0. \)

While the increase in labor demand for the cash constrained firms is clear, labor demand for the cash unconstrained firms depends on specific assumptions on the consumers’ preferences, which in turns determines the equilibrium effects on the unconstrained firms’ production, wage and prices.

For the case that employment for the cash unconstrained firms decreases, then the equilibrium quantity of goods produced by the cash unconstrained firms decreases. This is because the monetary transfers lead the consumers to the decision to work less for any given wage. Labor supply decreases, increasing the wage. Then, cash unconstrained firms increase their price and decrease their production.

5 Optimal Monetary Policy

We ask the question of how monetary policy should optimally respond to productivity shocks in our model, which includes both cash constrained and cash unconstrained firms. The answer relates with the findings of the previous two sections, where we show that i) a positive productivity shock affects the two types of firms differently, i.e., increases labor hired by the cash unconstrained firms and decreases the labor hired by the constrained firms and ii) expansionary monetary policy increases the employment and production of the cash constrained firms.

We will slightly change the timing of the model, in order to introduce monetary authority’s reaction to the productivity shock. The household will again allocate funds for consumption and loans before observing the productivity shock and monetary authority’s response. Then, the choice of \( x \) and \( n \) is made in the same way as it was made before. The household believes that monetary authority reacts to the productivity shock in a precise way, given by the function \( \mu(\theta) \). We will allow time between the realization of the productivity shock and monetary authority’s response.

In order to employ optimal monetary policy, we first study the social planner’s problem. The planner would choose consumption of the the two goods, \( c^1, c^2 \geq 0 \) and labor supplied
\( H \in [0, 1] \), in order to maximize equation (1), under constraints (26), (27) and (28). The solution implies that:

\[
U'(c^{1*}) = V'(c^{2*}) = \frac{D'(H^*)}{\theta},
\]  

(39)

from where we can infer that \( \frac{d c^{1*}}{d c^{2*}} > 0 \). Differentiating both sides with respect to \( \theta \) we find that \( \frac{d c^{1*}}{d \theta}, \frac{d c^{2*}}{d \theta} > 0 \).

Now we want to explore the model’s equilibrium given the efficient choices of \( c^{1*}, c^{2*}, H^* \).

From equations (13), (14) and (39) we have that:

\[
p^{1*} = p^{2*} = p^*. 
\]  

(40)

Unconstrained firms maximize profits as before, with maximum profits condition (21). Then, given (40) the wage is:

\[
w^* = p^* \theta. 
\]  

(41)

Given the maximization problem of the cash constrained firms, and the wage as specified above, it turns out that \( R^* = 0 \). That’s because it is efficient for all firms to have full access to the financial system and \( R^* = 0 \) ensures that both types of firms have. We will now find the monetary policy rule that will implement this equilibrium.

The market clearing conditions are the same as before for all markets, so equations (26), (27), (28), (29), (30), hold. Also, from equation (32) we have that:

\[
1 + \mu = \frac{\beta \psi w}{D'(H)},
\]

which using equation (31) becomes:

\[
1 + \mu = \frac{\beta \psi x}{\beta \psi - D'(H)H^c}.
\]

The above equation will help us specify the optimal monetary policy rule. Using equations (26) and (39) it becomes:

\[
1 + \mu^*(\theta) = \frac{\beta \psi}{\beta \psi - U'(c^{1*})c^{1*}}.
\]  

(42)

We see that the response of optimal monetary policy to the productivity shock depends
on the household’s response of labor supplied. Differentiating equation (42) with respect to the productivity shock we see that the optimal monetary policy’s response to a positive productivity shock is to expand, whenever the substitution effect dominates the income effect, i.e., whenever:

\[- \frac{U''(c^{1\ast})c^{1\ast}}{U'(c^{1\ast})} < 1.\]

Also, from equations (8) and (40) we can solve for the optimal price level:

\[ p^* = \frac{x}{c^{1\ast} + c^{2\ast}}, \quad (43) \]

which decreases with a higher productivity shock. We can also solve for the optimal wage level using equation (41):

\[ w^* = \frac{\theta x}{c^{1\ast} + c^{2\ast}}. \quad (44) \]

Then, the less money the household decides to spend for consumption, \( x \), the lower the price level and the wage. In addition, \( \frac{dw^*}{d\theta} > 0 \), so a positive productivity shock increases the optimal wage. Under optimal monetary policy, money supply increases after a positive productivity shock. With expansionary monetary policy the cash constrained firms receive extra liquidity to finance the higher wage and the asymmetry between the cash constrained and cash unconstrained firms disappears. Consumers prefer to work more, and they get employed by both the cash constrained and unconstrained firms.

6 Conclusion

We use a large family model, where consumption decisions are subject to a cash-in-advance constraint, and labor hiring decisions are subject to another cash-in-advance constraint, but only for a part of firms’ population, the firms that are cash constrained. Because of the dissimilarity in the way the two types of firms finance their operation, productivity and monetary policy shocks affect them differently. We find that a positive productivity shock increases labor hired by the cash unconstrained and reduces the labor hired by the cash constrained firms. This is because a positive productivity shock increases the competitive wage the firms offer in order to hire labor services. The higher wage is financed through
borrowing by the cash constrained firms exposing them to a negative effect. The cash unconstrained firms are not subject to the external finance premium and are not exposed to this negative effect.

Low productivity shocks have the opposite results. A decrease in productivity is bad news for both types of firms, who decrease their production. However, as the wage decreases, external financing becomes less of a burden for the cash constrained firms, who increase their employment. Our results concerning the effects of productivity shocks are consistent with the empirical findings of Moscarini and Postel-Vinay (2012).

Monetary policy shocks also affect differently the two types of firms. The cash constrained firms borrow to finance production and thus they are the ones to be directly affected by monetary policy actions. An increase in money supply increases the supply of loans to the cash constrained firms, increasing their employment and production. On the other hand, monetary policy tightening decreases the supply of loanable funds, decreasing the employment and production of the cash constrained firms. Our results concerning the effects of monetary policy shocks are consistent with the empirical findings of Gertler and Gilchrist (1994).

Combining the intuition of the two previous findings, monetary policy that follows the optimal rule attempts to facilitate the negative effects that productivity shocks have on the cash constrained firms; it increases the amount of funds available for borrowing in the loanable funds market after a positive productivity shock. Optimal monetary policy tights and decreases these funds after a negative productivity shock, when the wage is low and the external premium burden for the cash constrained firms is low. The result suggests that optimal monetary policy reacts procyclically to the productivity shock, in order to balance labor flows between the cash constrained and the unconstrained firms.

Our results are consistent with empirical findings (Moscarini and Postel-Vinay, 2012; Gertler and Gilchrist, 1994) if we let small firms be a proxy for cash constrained firms and large firms be a proxy for cash unconstrained firms. However, we need to note that these empirical findings include a mix of productivity shocks, monetary policy shocks and credit conditions shocks. Our work motivates future empirical work that takes into account the different shocks and includes proxies for credit conditions and monetary policy decisions,
when studying the effects of productivity shocks on different types of firms.
References


