Unemployment and Productivity in the Long Run: the Role of Macroeconomic Volatility

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Unemployment and its low frequency component

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Unemployment trend and productivity growth trend

5 year rolling windows

Unemployment Trend (%)

Productivity Growth Trend (%)

VAR estimates

Unemployment Trend (%)

Productivity Growth Trend (%)

Pierpaolo Benigno, Luca Ricci, Paolo Surico
Unemployment trend and productivity growth trend

**OLS ESTIMATES**

\[
\tilde{u}_t = 0.10 - 2.24 \cdot \tilde{g}_t + \hat{e}_t
\]

with \( R^2 = 0.77 \).

**Empirical works:**


**Theoretical works:**

- on labor supply: Ball and Mankiw (2002), Ball and Moffitt (2002).
But...

- In the 80s, productivity growth cannot explain a large portion of the fall in long-run unemployment. (The Great Moderation)
- In the early 90s, a flat productivity growth cannot explain the fall in long-run unemployment.
- Since the early 2000s, an increase in productivity growth comes with a puzzling rise in long-run unemployment.
Unemployment trend and productivity growth volatility

VAR estimates

5 year rolling windows

Unemployment Trend
Productivity Growth Variance

Unemployment Trend
Productivity Growth Variance

Pierpaolo Benigno, Luca Ricci, Paolo Surico
OLS ESTIMATES

\[ \tilde{u}_t = 0.08 - 1.68 \cdot \tilde{g}_t + 50.89 \cdot \tilde{\sigma}^2 + \tilde{\epsilon}_t \]

where \( R^2 = 0.95 \)

- Back of the envelope calculations show that during the 80s a fall in the volatility of productivity contributed to more than 50% of the fall in long-run unemployment, while since 2000 the rise in the volatility contributed to more than 70% of the rise in long-run unemployment.

- No literature on this relationship!
Our explanation for joint role of productivity and volatility

- Real wage rigidities naturally imply a negative long-run relationship between unemployment and productivity growth.
- Asymmetric real wage rigidities strongly reinforce this relationship at low trends of productivity growth and can also account for the role of its volatility.

**Intuition**

\[
\text{Real Profits} = Y - \frac{W}{P}L = AL^\alpha - \frac{W}{P}L
\]

- When the productivity trend is low and real wage $W/P$ is stickier downward than upward, recessions are much worse and expansions are not better.
- When the trend in $A$ is flat and $W/P$ is stickier downward than upward, then higher volatility of $A$ leads to lower $L$ on average.
The model

- The economy is subject to an aggregate productivity shock $A_t$, whose logarithmic $a_t$ is distributed as a Brownian motion

$$da_t = g dt + \sigma dB_t$$

where $B_t$ denotes a standard Brownian motion with zero drift and unit variance.

- Household $j$ has preferences over time given by

$$E_{t_0} \left[ \int_{t_0}^{\infty} \exp{-\rho (t-t_0)} \left( \ln C_t - \frac{1^{1+\eta}(j)}{1 + \eta} \right) dt \right]$$

$\rho > 0$ is the rate of time preference. Standard intertemporal budget constraint and optimality conditions apply.
Firms

- Technology for the production of all goods
  \[ y_t(i) = A_t L_t(i)^\alpha, \]
  for a parameter \( \alpha \) with \( 0 < \alpha < 1 \).
- Prices are flexible and set in a monopolistic-competitive goods market. Optimality conditions implies
  \[ p_t(i) = P_t = \mu_p \frac{W_t L_t(i)}{Y_t} = \mu_p \frac{W_t L_t}{Y_t} \]
  where \( \mu_p \equiv \theta_p / [(\theta_p - 1)\alpha] > 1 \) denotes the mark-up.
- The demand for labor of type \( j \) is given by
  \[ l_t^d(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\theta_w} L_t \]
Labor supply

Given labor demand, wage setters set real wages to maximize present discounted value of the marginal utility of wage income minus the disutility of working. Equivalent formulation of the labor-supply problem is the maximization of the following objective

\[ E_{t_0} \left[ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} \pi(w_t(j), w_t, A_t) dt \right] \]

by choosing real wages \( \{w_t(j)\}_{t=t_0}^{\infty} \), where

\[ \pi(w_t(j), w_t, A_t) \equiv \]

\[ \frac{1}{\mu_p} \left( \frac{w_t(j)}{w_t} \right)^{1-\theta_w} - \frac{1}{1 + \eta} \left( \frac{1}{\mu_p} \right)^{\frac{1+\eta}{1-\alpha}} \left( \frac{w_t(j)}{w_t} \right)^{-(1+\eta)\theta_w} \left( \frac{A_t}{w_t} \right)^{\frac{1+\eta}{1-\alpha}} \]
Flexible wages

Static problem. Optimality condition:

\[ \pi_{w_j}(w_t(j), w_t, A_t) = 0 \]

Labor is constant

\[ L^f = (\mu_p \mu_w)^{-\frac{1}{1+\eta}} \]

Real wages are proportional to the aggregate productivity shock

\[ w_t^f = \frac{1}{\mu_p} (L^f)^{\alpha-1} A_t^\alpha \]
Unemployment

- As in Galí (2010), unemployment is defined as the difference between the “notional” amount of labor that workers would be willing to supply in a competitive and frictionless market at the current real wage and the amount of labor currently employed.

\[ u_t = \ln L_s^t - \ln L_t \]

- Notional labor supply defined as the amount of labor that equates the marginal rate of substitution between labor and (current) consumption at the current real wage

\[ (L_s^t)^\eta C_t = \frac{W_t}{P_t} \]

- Unemployment and output gap are related through

\[ u_t = u^f - \frac{1 + \eta}{\eta} x_t \]

where \( x_t \) is the output gap and \( u^f \) is the flexible-price-wage unemployment rate.
Sticky real wages

- Wage setters take into account the costs of changing real wages

\[
V(w_t(j), w_t, A_t) = \max_{\pi_{R,t}(j)} E_{t_0} \left[ \int_{t_0}^{\infty} e^{-\rho(t-t_0)} [\pi(w_t(j), w_t, A_t) - h(\pi_{R,t}(j))] dt \right]
\]

- Assume linear function for adjustment costs

\[
h(\pi_{R,t}(j)) = \frac{e^{\chi \lambda \pi_{R,t}(j)} - \chi \lambda \pi_{R,t}(j) - 1}{\lambda^2}
\]

for some parameters \(\chi, \lambda\), where real wage changes are

\(\pi_{R,t}(j) dt \equiv dw_t(j)/w_t(j)\).

- \(\chi\) is a measure of the costs of adjustment;
- \(\lambda\) measures the asymmetries in the cost function.
When $\lambda \to 0$, standard symmetric quadratic cost function

$$h(\pi_R, t(j)) = \chi^2 \frac{(\pi_R, t(j))^2}{2},$$

When $\lambda < 0$ it is more costly to adjust real wages downward than upward and vice versa when $\lambda > 0$.

When $\lambda \to -\infty$ real wages are inflexible downward and fully flexible upward.
• Optimality condition requires

\[ h_{\pi}(\pi_R, t(j)) = V_{w_j} w_t(j), \]

• Marginal costs of changing real wages follow a stochastic differential equations of the form

\[ \rho h_{\pi}(\pi_R, t) dt = \frac{\theta_w - 1}{\mu_p} \left[ \left( \frac{L_t}{L^f} \right)^{1+\eta} - 1 \right] dt + E_t dh_{\pi}(\pi_R, t) \]

• Under a quadratic cost function can be simplified to

\[ \rho \pi_{R, t} dt = k \left[ \left( \frac{L_t}{L^f} \right)^{1+\eta} - 1 \right] dt + E_t d\pi_{R, t} \]
Let $x_t \equiv \ln L_t - \ln L^f$ be the employment gap; $x_t$ follows diffusion process

$$dx_t = \frac{1}{1 - \alpha} (g - \pi_R(x_t)) \, dt + \frac{1}{1 - \alpha} \sigma dB_t,$$

which can be used to derive long-run distribution and in particular the long-run mean of the employment gap, $x$, if it exists. where

$$\pi_R(x_t) = \frac{\ln[1 + \lambda \chi p(x_t)]}{\chi \lambda}.$$

and $p(x_t)$ satisfies the following differential equation

$$\rho p(x_t) = k \left[ e^{(1+\eta)x_t} - 1 \right] + \frac{1}{1 - \alpha} p_x(x_t) (g - \pi_R(x_t))$$

$$+ \frac{1}{2} \frac{1}{(1 - \alpha)^2} p_{xx}(x_t) \sigma^2.$$
Theoretical results

1. **Symmetric adjustment costs:** negligible long-run trade-off between unemployment and productivity growth and marginal role for volatility in shifting the trade off.

2. **Asymmetric adjustment costs:** stronger trade-off and important role for volatility, the stronger the asymmetries in real wage adjustments.
Figure: long-run trade-off in the SYMMETRIC case
Figure: long-run trade-off in the ASYMMETRIC case
Limiting case: downward real wage rigidity

- As in the previous problem, but now real wages cannot fall and can freely move upward

\[ dw_t(j) \geq 0, \]

- Long-run mean of unemployment is given by

\[
E[u_\infty] = u^f + \frac{1}{2} \frac{1 + \eta}{\eta(1 - \alpha)} \frac{\sigma^2}{g} + \frac{1}{\eta} \ln c(g, \sigma^2, \eta, \rho, \alpha).
\]

for a function \(0 \leq c(g, \sigma^2, \eta, \rho, \alpha) \leq 1\).

- Under myopic adjustment rule, \(\rho \to \infty\), and

\[
E[u_\infty] = u^f + \frac{1}{2} \frac{1 + \eta}{\eta(1 - \alpha)} \frac{\sigma^2}{g},
\]
Figure: long-run trade-off in the INFLEXIBLE DOWNWARD case
Estimating long-run means and variances

- Consider a VAR

\[ Y_t = B_{0,t} + B_{1,t} Y_{t-1} + \ldots + B_{p,t} Y_{t-p} + \epsilon_t \equiv X_t' \theta_t + \epsilon_t \]

with drifting coefficients \( \theta_t \) and stochastic volatility \( \text{Var}(\epsilon_t) \equiv \Omega_t \)

- \( Y_t \equiv [g_t, \Delta w_t, u_t]' \), and \( p \) is set equal to 2.


- MCMC estimation method.
Let us rewrite VAR in companion form:

\[ z_{t|T} = C_{t|T} + D_{t|T} z_{t-1} + \zeta_t \]

The long-run mean of \( z_{t|T} \) can then be computed as:

\[ \tilde{z}_{t|T} = (I - D_{t|T})^{-1} C_{t|T} \]

where we use local-to-date \( t \) approximations to the mean of the endogenous variables evaluated at the posterior mean \( E(\theta_{t|T}) \).

The time-varying variance of \( z_{t|T} \) can be computed using the integral of the spectral density over all frequencies, \( \int_\omega f_{t|T}(\omega) \), where

\[ f_{t|T}(\omega) = (I - D_{t|T} e^{-i\omega})^{-1} \frac{\Omega_{t|T}}{2\pi} \left[ (I - D_{t|T} e^{-i\omega})^{-1} \right]' \]
Evaluating the model

- Model implies

\[ E[u_\infty] = f(g, \sigma^2, \theta) \]

for a vector of parameters \( \theta \)

- OLS estimates of reduced-form linear model

\[ \tilde{u}_t = 0.10 - 2.24 \cdot \tilde{g}_t + \hat{\epsilon}_t \]

with \( R^2 = 0.77 \).

- OLS estimates of linear model on mean and variance of productivity growth

\[ \tilde{u}_t = 0.08 - 1.68 \cdot \tilde{g}_t + 50.89 \cdot \tilde{\sigma}_t^2 + \hat{\epsilon}_t \]

where \( R^2 = 0.95 \).
Literature has argued that changes in the demographic composition of the labour force affects the low-frequency movements in unemployment (Shimer, 1998), the low-frequency movements in productivity (Francis and Ramey, 2009) and the variance of real output growth (Jaimovich and Siu, 2009).
Controlling for demographics

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- Control for demographics:
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- Control for demographics:
  - construct time series for the share of workers in the labor force with age (i) between 16 and 21, (ii) between 16 and 34, and (iii) the sum of the shares of workers in the 16-29 and the 60-64 windows of age.
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- Control for demographics:
  - construct time series for the share of workers in the labor force with age (i) between 16 and 21, (ii) between 16 and 34, and (iii) the sum of the shares of workers in the 16-29 and the 60-64 windows of age.
  - run a regression of the unemployment rate on a constant and the unemployment rate of workers in prime age (defined as those between 35 and 64 years) to construct a measure of genuine unemployment (to use in the VAR) which is not affected by demographics.
Table 1: Controlling for demographics

<table>
<thead>
<tr>
<th>specifications:</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<td>16-21</td>
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<td>✓</td>
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<td>✓</td>
<td>✓</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
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</tr>
</tbody>
</table>

| regressors: | constant | 0.092*** | 0.095*** | 0.076*** | 0.085*** | 0.076*** | 0.082*** | 0.076*** | 0.055*** |
|             |         | (0.002) | (0.005) | (0.004) | (0.007) | (0.001) | (0.006) | (0.002) | (0.045) |
|             | \bar{g}_t | -2.014*** | -2.023*** | -1.717*** | -1.252*** | -1.679*** | -1.824*** | -1.685*** | -0.680*** |
|             |         | (0.050) | (0.150) | (0.064) | (0.127) | (0.047) | (0.071) | (0.053) | (0.125) |
|             | labor force share_t | 0.060*** | 0.011* | 0.044*** | -0.005 | -0.008*** | -0.001 |
|             |         | (0.007) | (0.006) | (0.005) | (0.005) | (0.005) | (0.005) |
|             | \bar{g}_t^2 | 52.37*** | 52.20*** | 51.41*** | 57.21*** |
|             |         | (2.482) | (1.997) | (2.512) | (6.714) |
| \hat{R}^2   |         | 0.832 | 0.776 | 0.838 | 0.934 | 0.949 | 0.950 | 0.949 | 0.515 |
Limiting model with downward real wage rigidity implies

\[ \tilde{u}_t = u^f + \frac{1}{2} \frac{1 + \eta}{\eta (1 - \alpha)} \tilde{\sigma}_t^2 + \frac{1 + \eta}{\eta} \ln c(\tilde{g}_t, \tilde{\sigma}_t^2, \eta, \rho, \alpha) + \epsilon_t. \]

With myopic adjustments in real wages, \( \rho \to \infty \):

\[ \tilde{u}_t = u^f + \frac{1}{2} \frac{1 + \eta}{\eta (1 - \alpha)} \frac{\tilde{\sigma}_t^2}{\tilde{g}_t} + \epsilon_t \]
International evidence

- Our international dataset is an unbalanced panel of quarterly observations for developed and developing economies over the post-WWII period.
- For each country \( i \), we compute over a window of ten years:
  - (i) the mean of unemployment, \( \tilde{u}_{it} \),
  - (ii) the mean of productivity growth, \( \tilde{g}_{it} \),
  - (iii) the variance of productivity growth, \( \tilde{\sigma}^2_{it} \), and
  - (iv) the ratio between the variance of productivity growth and the mean of productivity growth, \( V\text{-to-}M \text{ ratio}_{it} \).

Results:

- confirm role of productivity and especially volatility,
- mainly a time series effect (rather than cross-sectional).
### International evidence

<table>
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<th>estimation method:</th>
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<tr>
<td>$\tilde{g}_t$</td>
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<td>-0.561***</td>
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<td>$\tilde{\sigma}^2_t$</td>
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<td>$\tilde{\sigma}^2_t / \tilde{g}_t$</td>
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<td>$R^2$</td>
<td>0.045</td>
<td>0.120</td>
<td>0.223</td>
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Table: 

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<td>$R^2$</td>
<td>0.357</td>
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Conclusions

- Productivity growth and unemployment trends are negatively related.
- New striking evidence for an important role of the volatility of productivity growth in shifting the long-run trade off.
- A model with symmetric real wage rigidities can barely account for the first empirical finding.
- A model with asymmetric real wage rigidities can account for both empirical results.