ORANI-IT: a Computable General Equilibrium Model of Italy

Francesco Felici, Maria Gesualdo
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ORANI-IT: a Computable General Equilibrium Model of Italy

Francesco Felici¹, Maria Gesualdo²

Abstract

This paper presents the comparative-static national disaggregate Computable General Equilibrium (CGE) model of the Italian economy, ORANI-IT, which represents the starting point to the development of a tax CGE model of Italy. The model, designed at the Department of Treasury of the Italian Ministry of the Economy and Finance, in collaboration with the Centre of Policy Studies (CoPS), and currently managed at Sogei S.p.A. (IT Economia - Modelli di Previsione ed Analisi Statistiche), is intended for policy analysis. The aim of this paper is to provide a complete description of the theoretical specification of the model and to illustrate the process of compiling the model’s database. Departing from the core structure, features of the Italian model in the context of ORANI-style models, developed at CoPS, are highlighted, as data availability allowed us to improve the modelling of the investment matrix and the demand of labour. The result is a more reliable model with a broader level of analysis. The paper concludes with the model’s validation, which among several checks, consists in an illustrative two targets/two instruments simulation.

JEL Classification: C68, E10, E60
Keywords: Computable general equilibrium (CGE) model, Model’s validation, Italy

¹ Ministero dell’Economia e delle Finanze, Dipartimento del Tesoro. E-mail: francesco.felici@tesoro.it
² Sogei S.p.A., IT Economia – Modelli di previsione ed analisi statistiche Unit. E-mail: maria.gesualdo@imtlucca.it
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1 Introduction

CGE modelling started with the seminal work of Leontief in the 1930's with the setting up of a Table of Input-output accounts for the US economy; considered a prototype of CGE. In 1960 Leif Johansen developed the first empirically based, multi-sector, price endogenous model, aimed at analysing resources allocation issues.

Johansen invented a method of economic analysis through a combination of the national accounts, macroeconomic balancing equations and input output analysis. He used the Input-Output method to create a data set for the economy at one specific point in time. Then, after having assigned production functions and demand functions to producers and consumers, he found the equilibrium of the theoretical model in the macro balancing equations, given by the national accounting framework.

Along with CGE, Herbert Scarf's algorithm (1967) for the numerical determination of the equilibrium of a Walrasian system represents the pioneer work for Applied General Equilibrium (AGE) models. However, because Scarf's method was proven non-computable to a precise solution, AGE models were surpassed by CGE models in the middle 1980s.

In the general equilibrium tax literature, the path breaking Harberger's model (1962) for tax policy analysis, regarding the incidence and efficiency effects of taxes, has been a major stimulus to subsequent works. Harberger (1964) develops a procedure for estimating the welfare cost of a distortionary factor tax, presenting the generalized triangle formula. Shoven and Whalley (1973) incorporate taxes into a multi-sectoral general equilibrium framework, to analyse the impact of government tax policy. Ballard, Scholz and Shoven (1987) with the aim of assessing alternative fiscal designs, extend the general equilibrium framework for tax policy evaluation previously developed by Ballard et al. (1985), via the inclusion of a Value-Added-Tax framework.

The contribution this paper makes is to fill the existing gap in the Italian CGE literature, as there has been little research on this topic. Ciaschini et al. (2012) build a bi-regional CGE model based on a bi-regional Social Accounting Matrix for Italy for 2003, aimed at verifying the impact of an environmental fiscal reform. The model refers to previous studies by Ciaschini and Socci (2007), and by Fiorillo and Socci (2002). Standardi, Bosello, Eboli (2013) present a sub-national version of the GTAP model for Italy, with the aim of assessing climate change impacts. Our work is from a different prospective, as it is intended as a tool for industrial and fiscal analysis.

In this paper a national CGE model for Italy, ORANI-IT, is presented. The model is designed for the Ministry of Economy and Finance, in collaboration with the Centre of Policy Studies, and is intended to provide practical policy recommendation. The aim is to enhance the analysis capability of the Ministry of Economy and Finance, with a particular focus on the assessment of industrial and fiscal policies.

The theoretical structure and computer code of the Italian model closely follow that of ORANI, model developed at the Centre of Policy Studies (CoPS), fully documented in Dixon et al. (1982) and Horridge (2003). The ORANI model and many CGE models based on its theoretical structure have been widely used as tools for practical policy analysis by academics and researchers in public and private sectors in Australia and in many other countries. Although ORANI represents the skeleton of the core model, we adapt and extend its theory to better meet the features of the Italian economy and the requirements of a given economic research question. In detail, we improve the reliability of the model's database, by replacing assumptions commonly used in the construction of ORANI-style models with actual data provided by ISTAT; and expand the model's theory by further specifying the demand for labour, via the inclusion of additional nests to the original production structure.
The paper is structured as follows. Section 2 describes the theory of the model. It consists of: an outline of the structure of the model and of the appropriate interpretation of the results of comparative-static simulations; and a complete description of the theoretical specification of the model framed around the TABLO input file which implements the model in the GEMPACK suite of software programs (Harrison and Perason 1996). The complete system of equations is given in the TABLO Appendix. Section 3 describes the process of compiling the model’s database. Section 4 focuses on the features of the Italian model. Section 5 presents the model’s validation, which among several checks, consists in an illustrative two targets/two instruments simulation. Section 6 concludes, followed by an appendix presenting a miscellaneous of formal derivations.

2 Theoretical framework

General equilibrium models are built on two essential ingredients: optimization behaviours and conditions for equilibrium. The theoretical model is structured in terms of behavioural equations and identities, which give an algebraic representation of the Arrow-Debreu general equilibrium structure. Behavioural equations feed into the functional forms, which in turn equal under the macro balancing equations. The theoretical model feeds into the database, which is built on national accounts data, and that comes to represent the benchmark economy.

2.1 Model structure overview

ORANI-IT has a theoretical structure that consists of equations describing, for some time period:

• producers' demands for produced inputs and primary factors;
• producers' supplies of commodities;
• demands for inputs to capital formation;
• household demands;
• export demands;
• government demands;
• the relationship of basic values to production costs and to purchasers' prices;
• market-clearing conditions for commodities and primary factors; and
• numerous macroeconomic variables and price indices.

The behaviour of the agents stems from conventional neoclassical microeconomics, with demand and supply equations deriving from solutions to the optimisation problems (cost minimisation, utility maximisation, etc.). The agents are assumed to be price-takers, with producers operating in competitive markets which prevent the earning of pure profits.

2.2 The percentage-change approach to model solution

The theoretical model is represented via a system of non-linear equations. Following Johansen (1960), the model is solved by representing it as a series of linear equations relating percentage changes in model variables. This system is then easily solved by numerical integration technique, such as Euler procedure. Appendix A explains how the linearised form can be used to generate exact solutions of the underlying non-linear equations, as well as to compute linear approximations to those solutions.

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3 For a detailed treatment of the linearised approach to CGE modelling, see Dixon et al. (1992). Chapter 3 contains information about Euler's method and multistep computations.
2.2.1 The initial solution

The solution procedure to work requires an initial solution of the model.

The present model is calibrated on the Supply and Use Tables (SUT) of the Italian economy for the year 2008 released by ISTAT\(^4\), so that the model features 63 industries, 63 commodities, 63 investors, a representative household, a government, and an export sector.

Generally, the initial solution is built on the most recently historical data available. However, this is not the case for the Italian model, which despite the availability of the SUT for 2009, relies on 2008 data. Considering the statement of fact that the year 2009 has been characterized for being particularly harsh under an economic point of view, as mentioned by the IMF in the World Economic Outlook for 2009 and confirmed by ISTAT in the Rapporto Annuale 2009, the 2008 SUT is preferred so to build a more representative baseline scenario.

2.3 The model database

Figure 1 gives a schematic representation of the model's input-output database, revealing the basic structure of the model. Three main parts are set out: an absorption matrix, a production matrix, and a trade tax matrix. The column headings in the absorption matrix identify the following demanders:

- domestic producers divided into I industries;
- investors divided into I industries;
- a single representative household;
- an aggregate foreign purchaser of exports;
- government demands; and
- changes in inventories.

The entries in each column show the structure of the purchases made by the agents identified in the column heading. Each of the C commodity types identified in the model can be obtained locally or imported from overseas. The source-specific commodities are used by industries as inputs to current production and capital formation, are consumed by households and governments, are exported, or are added to or subtracted from inventories. Only domestically produced goods appear in the export column. M of the domestically produced goods are used as margins services (wholesale and retail trade, and transport) which are required to transfer commodities from their sources to their users. Commodity taxes are payable on the purchases. As well as intermediate inputs, current production requires inputs of three categories of primary factors: labour (divided into O occupations), fixed capital, and agricultural land. Production taxes include output taxes or subsidies that are not user-specific. The 'other costs' category covers various miscellaneous taxes on firms, such as municipal taxes or charges.

Each cell in the absorption matrix contains the name of the corresponding data matrix. For example, V2MAR is a 4-dimensional array showing the cost of M margins services on the flows of C goods, both domestically produced and imported (S), to I investors.

The MAKE matrix at the bottom of Figure 1 shows the value of output of each commodity by each industry. In principle, each industry is capable of producing any of the C commodity types. Finally, tariffs on imports are assumed to be levied at rates which vary by commodity but not by user. The revenue obtained is represented by the tariff vector V0TAR.

\(^4\) Downloaded at [http://www.istat.it/it/archivio/60913](http://www.istat.it/it/archivio/60913)
Each of the flow in Figure 1 is the product of a price and a quantity.

The first row in the absorption matrix (the “BAS matrices”: V1BAS,…,V6BAS) shows flows in year $t$ of commodities to each user. Each of these matrices has $C \times S$ rows, one for each of $C$ commodities from $S$ sources. In ORANI-IT, $C$ is 63 and $S$ is 2 (domestic and imported). The flows are valued at basic prices. The basic price of a domestically produced good is the price received by the producer (that is, the price paid by users excluding sales taxes, transport costs and other margin costs). The basic price of an imported good is the landed-duty-paid price, i.e. the price at the port of entry just after the commodity has cleared customs.
The second row (the “MAR matrices”: V1MAR,…,V5MAR) shows the values of margin services used to facilitate the flows of commodities identified in the BAS matrices. The commodities used as margins are domestically produced trade, road transport, rail transport, water transport and air transport services. The model assumes that margin services are domestically produced and are valued at basic prices. Imports are not used as margin services. Each of the margin matrices has CxSxM rows. These correspond to the use of M margin commodities in facilitating flows of C commodities from S sources. Inventories (column 6) are assumed to comprise mainly of unsold products, and therefore do not bear margins. As with the BAS matrices, all the flows in the MAR matrices are valued at basic prices. Consistent with the UN convention (UN 1999:33), we assume that there are no margins on services.

The costs of margins services, together with indirect taxes, account for differences between basic prices (received by producers or importers) and purchasers' prices (paid by users).

The third row (the “TAX matrices”: V1TAX,…,”V5TAX) shows sales taxes on flows to different users. ORANI-IT treats commodity taxes in great detail. The tax levied on each basic flow is separately identified, and the tax rates can differ between users and between sources. The disaggregated structure allows to simulate the effects of commodity-and-user-specific tax changes, such as an increased tax on Cokes and refined petroleum used by the Land transport industry. In such a way, tax rates on a commodity used as an intermediate input to producers can divert from that on household consumption of the same commodity.

Besides intermediate inputs, current production requires inputs of three types of primary factor: labour (divided into occupations), fixed capital, and agricultural land. These are shown in rows 4,5 and 6. Industries also have to pay production taxes and other cost tickets (rows 7-8). The ‘other costs’ category covers various miscellaneous costs for firms, such as municipal taxes or charges, or the costs of holding inventories.

MAKE is a CxI matrix showing the value of commodity c∈COM produced by industry i∈IND. In principle, an industry can produce more than one commodity, and a commodity can be produced by more than one industry. Indeed, that is a feature of the MAKE matrix for Italy.

There are three basic conditions that the database must satisfy. First, industry costs should equal industry sales. This is necessary to satisfy the model’s zero pure profit assumption. Second, the value of the total output of a given commodity should equal the value of the total usage of that commodity. This is necessary to satisfy the model’s commodity market clearing assumption. Finally, because of the condition that flows of goods, services and factors cannot be negative, flows matrices should not contain negative numbers, except for matrices related to taxes and inventories (VTAXs and V6BAS).

### 2.4 The equation system

ORANI-IT allows each industry to produce several commodities, using as inputs domestic and imported commodities, labour of several types, land, and capital. In addition, commodities destined for export are distinguished from those for local use. The multi-input, multi-output production specification is kept manageable by a series of separability assumptions, illustrated by the nesting shown in Figure 2.
2.4.1 Structure of production

The assumption of input-output separability implies that the generalised production function for some industry:

\[ F(\text{inputs}, \text{outputs}) = 0 \]  

may be written as:

\[ G(\text{inputs}) = X1TOT = H(\text{outputs}) \]  

where X1TOT is an index of industry activity.

Assumptions of this type reduce the number of estimated parameters required by the model.
Figure 2 shows that the H function in (2) is derived from two nested CET (constant elasticity of transformation) aggregation functions, while the G function is broken into a sequence of nests. At the top level, commodity composites, a primary-factor composite and ‘other costs’ are combined using a Leontief production function. Consequently, they are all demanded in direct proportion to X1TOT. Each commodity composite is a CES (constant elasticity of substitution) function of a domestic good and the imported equivalent. The primary-factor composite is a CES aggregate of land, capital and composite labour. Composite labour is a CES aggregate of occupational labour types. Although all industries share this common production structure, input proportions and behavioural parameters may vary between industries.

### 2.4.2 Demands for primary factors

![Diagram of demand for different types of labour](image)

**Figure 3. Demand for different types of labour**

For each industry i, the equations determining the occupational composition of labour demand in each industry are derived from the following optimisation problem:

Choose inputs of occupation-specific labour, $X1LAB(i,o)$, to minimize total labour cost, 

$$\text{Sum}\{o, OCC, P1LAB(i,o) \times X1LAB(i,o)\},$$

where

$$X1LAB\_O(i) = \text{CES}[\text{All},o, OCC: X1LAB(i,o)],$$

regarding as exogenous to the problem

$$P1LAB(i,o)$$ and $$X1LAB\_O(i).$$

The notation CES[ ] represents a CES function defined over the set of variables enclosed in the square brackets. Note that the problem is formulated in the levels of the variables. The solution of this problem, in percentage-change form, is presented below. Readers can refer to Appendix B for its derivation.

$$x1lab(i,o) = x1lab\_o(i) - \sigma_{lab}(i) \times [p1lab(i,o) - p1lab\_o(i)]$$  \hspace{1cm} (3)

Equation 3 indicates that demand for labour type o is proportional to overall labour demand, $x1lab\_o(i)$, and to a price term. In change form, the price term is composed of an elasticity of substitution, $\sigma_{lab}(i)$, multiplied by the percentage change in a price ratio $[p1lab(i,o) - p1lab\_o(i)]$ representing the wage of occupation o relative to the average wage for labour in industry i. Changes in the relative prices of the occupations induce substitution in favour of relatively cheapening
occupations. The percentage change in the average wage, $p_{1lab_o(i)}$, is given by the following equation:

$$p_{1lab_o(i)} = \sum{o, OCC, S1LAB(i,o) \cdot p1lab(i,o)}, \quad (4)$$

where $S1LAB(i,o)$ are the value shares of occupation $o$ in the total wage bill of industry $i$. In other words, $p_{1lab_o(i)}$ is a Divisia index of the $p1lab(i,o)$. It is worth noting that if the individual equations were multiplied by corresponding elements of $S1LAB(i,o)$, and then summed together, all price terms would disappear, giving:

$$x_{1lab_o(i)} = \sum{o, OCC, S1LAB(i,o) \cdot x1lab(i,o)} \quad (5)$$

This is the percentage-change form of the CES aggregation function for labour.

Turning to the demand for primary factors, its derivation follows a similar pattern to that underlying the previous nest.

In this case, total primary factor costs are minimised subject to the production function:

$$X1PRIM(i) = CES \left[ \frac{X1LAB_O(i)}{A1LAB_O(i)} \cdot \frac{X1CAP(i)}{A1CAP(i)} \cdot \frac{X1LND(i)}{A1LND(i)} \right] \quad (6)$$

Because we may wish to introduce factor-saving technical changes\(^5\), we include explicitly the coefficients $A1LAB_O(i)$, $A1CAP(i)$, and $A1LND(i)$.

From the system of first order differential equations it follows that demand for each primary factors, $x(i)$, is expressed in terms of percentage changes. As a rule:

$$x(i) - a(i) = x_{prim} - \sigma_{prim}(i)[p(i) + a(i) - \sum S(i)p_{prim}(i)] \quad (7)$$

From the homogeneity of degree zero of the demand for factors of production in the price vector follows that only relative prices are determined in equilibrium, and demand for each factor depends on the overall factor demand and on a price term. In change form, the price term is an elasticity of substitution, $\sigma_{prim}$, multiplied by the percentage change in a price ratio $[p(i) + a(i) - \sum S_{prim}(i)p_{prim}(i)]$ representing the cost of an effective unit of factor relative to the overall effective cost of primary factors to industry $i$. This implies that changes in the relative prices of the primary factors induce

\(^5\) A basic reference on facto-augmenting technical change is Allen (1967).
substitution in favour of relatively cheapening one. The percentage change in the average effective
cost, \(p1_{\text{prim}}(i)\), is a cost-weighted Divisia index of individual prices and technical changes.

Appendix B presents a formal derivation of CES demand equations with technical-change terms.

2.4.3 Sourcing of intermediate inputs

We adopt the Armington (1969; 1970) assumption that imports are imperfect substitutes for domes-
tic supplies in determining the import/domestic composition of intermediate commodity demands.
The total cost of imported and domestic good \(i\) are minimised subject to the production function:

\[
X1_C(c) = CES[All, s, SRC, x1(c,s,i)]
\]  

The result again present the same structure presented before, reflecting the CES composition. Commodity demand from each source is proportional to demand for the composite, and to the
elasticity of substitution between different sources, multiplied by the percentage change in a price ratio representing the effective price from the source relative to the effective cost of the
import/domestic composite. Lowering of a source-specific price, relative to the average, induces substitution in favour of that source. The percentage change in the average effective cost is again a
cost-weighted Divisia index of individual prices and technical changes.

\[
\text{Figure 5. Intermediate input sourcing decision}
\]

2.4.4 Top production nest

The topmost input-demand nest combine commodity composites, the primary-factor composite and
'other costs' using a Leontief production function, given by:

\[
X1TOT(i) = \frac{1}{A1TOT(i)} \times \text{MIN}\left[\text{All, c, COM: } \frac{x1S(c,i)}{A1S(c,i)} + \frac{x1PRIM}{A1PRIM} + \frac{x1OC(i)}{A1OC(i)}\right]
\]  

Consequently, each of these three categories of inputs identified at the top level is demanded in
direct proportion to the overall demand by industry, X1TOT(i). The Leontief production function is
equivalent to a CES production function with the substitution elasticity set to zero. Hence, the
demand equations resemble those derived from the CES case but lack price (substitution) terms. The $a_{1\text{tot}}(i)$ are Hicks-neutral technical-change terms, affecting all inputs equally.

2.4.5 Industry costs and production taxes

The model contains equations computing levels and changes in the total cost of production ad valorem production tax. The percentage change in the unit cost of production for industry $i$, $p_{1\text{tot}}(i)$, becomes also the percentage change in marginal cost, because of the constant returns to scale which characterise the model's production technology. We enforce the competitive Zero Pure Profits condition (price = marginal cost) by assuming that the $p_{1\text{tot}}$ are also equal to the average price received by each industry. Under constant returns to scale, we deduce:

$$p_{1\text{tot}}(i) = \sum S_k p_k + \text{tax and technical change terms}$$

where $S_k$ and $p_k$ are respectively the share of input $k$ in total cost, and the percent change in its price.

2.4.6 From industry outputs to commodity outputs

ORANI-IT allows for each industry to produce a mixture of all the commodities. For each industry, the mix varies, according to the relative prices of commodities. Here, the total revenue from all outputs is maximised subject to the production function:

$$X_{1\text{TOT}}(i) = \text{CET[ All,c,COM: Q1(c,i)]}$$

(9)

The CET (constant elasticity of transformation) aggregation function is identical to CES, except that the transformation parameter in the CET function has the opposite sign to the substitution parameter in the CES function. An increase in a commodity price, relative to the average, induces transformation in favour of that output. The average unit revenue is the same of the effective price of a unit of activity, and the Zero Pure Profits condition holds in the output market as well.

---

Figure 6. Composition of output

---

6 Multiproduction can be useful even where each industry produces just one commodity. For example we could split electricity generation into 2 parts: oil-fired and nuclear, each producing the same commodity, electricity.
All industries producing, say, Cereals, receive the same unit price. Cereals produced by different industries are deemed to be perfect substitutes\(^7\). All industries’ output of each commodity are simply add up to get the total supply.

2.4.7 **Export and local market versions of each good**

The model allows for the possibility that goods destined for export are not the same as those for local use. Conversion of an undifferentiated commodity into goods for both destinations is governed by a CET transformation frontier.

2.4.8 **Demands for investment goods**

Figure 7 shows the nesting structure for the production of new units of fixed capital. Capital is assumed to be produced with inputs of domestically produced and imported commodities. The production function has the same nested structure as that which governs intermediate inputs to current production. No primary factors are used directly as inputs to capital formation.

![Figure 7. Structure of Investment Demand](image)

The investment demand equations are derived from the solutions to the investor's two-part cost-minimisation problem. At the bottom level, the total cost of imported and domestic good \(i\) is minimised subject to the CES production function:

\[
X_2 S(c,i) = \text{CES}[\text{All}, s, \text{SRC: } \frac{X_2(c,s,i)}{A_2(c,s,i)}],
\]

\(^7\) The perfect substitute assumption causes problems when we have two industries just producing the same commodity, and neither industry has a fixed factor; e.g., in a long-run closure with mobile capital, and two electricity industries (say, nuclear and coal-fired) both making electric power. The model will find it hard to decide what proportion of generation should be nuclear. To cover such a case, the TABLO input file contains optional equations (not shown here), which allow (for example) nuclear electricity to be an imperfect substitute for coal-fired electricity.
while at the top level the total cost of commodity composites is minimised subject to the Leontief production function:

\[
X2TOT(i) = \frac{1}{A2TOT(i)} \text{MIN} \{ \text{All, c, COM} : \frac{X2_S(c,t)}{A2_S(c,t)} \} \tag{11}
\]

The total amount of investment in each industry, X2TOT(i), is exogenous to the above cost-minimisation problem. It is determined by other equations, covered later on.

Equation describing the demand for source-specific inputs follow the same structure as the corresponding intermediate demand equations presented previously. Also included is an equation which determines the price of new units of capital as the average cost of producing a unit—a Zero Pure Profits condition.

2.4.9 Household demands

As Figure 8 shows, the nesting structure for household demand is nearly identical to that for investment demand. The only difference is that commodity composites are aggregated by a Klein-Rubin, rather than a Leontief, function, leading to the linear expenditure system (LES). The equations for the lower, import/domestic nest are similar to the corresponding equations for intermediate and investment demands.

The allocation of household expenditure between commodity composites is derived from the Klein-Rubin utility function:

\[
\text{Utility per household} = \prod_{c} \left( X3_S(c) - X3SUB(c) \right)^{S3LUX(c)}, \tag{12}
\]

The X3SUB and S3LUX are behavioural coefficients—the S3LUX must sum to unity. Q is the number of households. The demand equations that arise from this utility function are:

\[
X3_S(c) = X3SUB(c) + S3LUX(c) * V3LUX_C/P3_S(c), \tag{13}
\]

where:

\[
V3LUX_C = V3TOT - \sum X3SUB(c) * P3_S(c) \tag{14}
\]

The name of the linear expenditure system derives from its property that expenditure on each good is a linear function of prices (P3_S) and expenditure (V3TOT). The form of the demand equations gives rise to the following interpretation. The X3SUB are said to be the 'subsistence' requirements of each good—these quantities are purchased regardless of price. V3LUX_C is what remains of the consumer budget after subsistence expenditures are deducted—we call this 'luxury' or 'supernumerary' expenditure. The S3LUX are the shares of this remnant allocated to each good—the marginal budget shares. Such an interpretation facilitates our transition to percentage change form, which begins from the levels equations:

\[
X3_S(c) = X3SUB(c) + X3LUX(c) \tag{15}
\]

\[
X3LUX(c) * P3_S(c) = S3LUX(c) * V3LUX_C \tag{16}
\]

\[
X3SUB(c) = Q * A3SUB(c) \tag{17}
\]

As equation (15) makes plain, the X3LUX are luxury usages, or the difference between the subsistence quantities and total demands. Equation (16) states that luxury expenditures follow the marginal budget shares S3LUX. Together, equations (15) and (16) are equivalent to (13). Equation (17) is necessary because our demand system applies to aggregate instead of to individual households. It states that total subsistence demand for each good c is proportional to the number of households, Q, and to the individual household subsistence demands, A3SUB(c).
The percentage-change form of the utility function (12), includes variables representing taste-changes, which allow the average budget shares to be shocked, in a way that preserves the pattern of expenditure elasticities. See Appendix C for further details.

The equations just described determine the composition of household demands but do not determine total consumption. That could be done in a variety of ways: set exogenously, determined by a consumption function, or determined via a balance of trade constraint.

### 2.4.10 Export demands

To model export demands, commodities in ORANI-IT are divided into two groups\(^8\). For an *individual export* commodity, foreign demand is inversely related to that commodity's price. For the remaining, *collective export*, commodities, foreign demand is inversely related to the average price of *all* collective export commodities.

The individual export group includes all the main export commodities. The model specifies downward-sloping foreign demand schedules for these commodities. In the levels, the equation would read:

\(^8\) The allocation of exports between collective and individual categories is driven by an indicator which measures the share of exports for each of the commodities on total production. Commodities whose ratio is above a certain threshold are classified as individual.
\[ X_4(c) = F_4Q(c) \left( \frac{P_4(c)}{PHI} \right)^{EXP\_ELAST(c)}, \]

where \( EXP\_ELAST(c) \) is a negative parameter—the constant elasticity of demand. That is, export volumes, \( X_4(c) \), are declining functions of their prices in foreign currency, \( (P_4(c)/PHI) \). The exchange rate \( PHI \) converts local to foreign currency units. The variables \( F_4Q(c) \) and \( F_4P(c) \) allow for horizontal (quantity) and vertical (price) shifts in the demand schedules.

The collective export group could include all those commodities for which the above equation is inappropriate, such sectors where export volumes do not seem to depend mainly on the corresponding price. The commodity composition of aggregate collective exports is exogenized by treating collective exports as a Leontief aggregate. Demand for the aggregate is related to its average price via a constant-elasticity demand curve, similar to those for individual exports.

### 2.4.11 Other final demands

ORANI does not provide any theoretical foundation for public demand. What the model does provide is a set of equations, which allow the public consumption to be exogenously determined, or alternatively, aggregate government consumption moves with real aggregate household consumption.

Turning to inventories, the percentage change in the volume of each commodity, domestic or imported, going to inventories, is the same as the percentage change in domestic production of that commodity.

### 2.4.12 Demands for margins

In the absence of technical change, demands for margins are proportional to the commodity flows with which the margins are associated.

### 2.4.13 Indirect taxes

ORANI-IT presents indirect taxes for producers, investors, households, and government. Sales taxes are treated as ad valorem on basic values of the corresponding commodity flows, and tax rates are powers of the sales-tax rates. Therefore, revenues are expressed as the product of the base and the power of the tax minus one, and the models tracks down changes in both the base and the rate. The model allows the changes in the relevant tax rates to be commodity-specific or user-specific.

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9 The ABS (absolute value) function is used to neutralize mistakes about the sign of \( EXP\_ELAST \).

10 The allocation of exports between collective and individual categories is driven by an indicator which considers the share of exports for each commodities on total production. Commodities whose ratio is above a certain threshold are classified as individual. Following this rule, the following commodities were classified as collective: Printing, Equipment repairs, Electricity and Gas, Water, Construction, Car Trade, Retail Trade, Post, Hotel and Restaurants, Financial Services, Real Estate, Legal Account, Public Administration, Education, Health, Social Works, Art Library and Bets, Sports Recreation, Active Member Organization, Repairs, Other Personal services, Households Own Production.
2.4.14 GDP from the income and expenditure sides

The model features a representation of the economic environment on both the demand and the supply side. GDP from the income side is made up by totals of factor payments, the value of other costs, and the total yield from commodity taxes. The GDP from the expenditure side gathers all the expenditure components, which are defined as a quantity index and a price index which sum to the nominal value of the aggregate. Note that our definition of real variables do not derive from the optimizing problems described before, but are constructed via price-quantity decomposition, as summary measures. Naturally, from the accounting identity, the measure from the income and expenditure side must equal.

2.4.15 The trade balance

Because zero is a plausible base-period value, the balance of trade is computed as an ordinary change, and as a fraction of the GDP to avoid choosing units.

2.4.16 Rates of return and investment

Turning to equations that determine the amount of new capital stock created for each industry, ORANI-IT contains 3 alternate investment rules, one of which will be active for each industry. The active rule either determines industry investment, or (if aggregate investment is fixed) it determines that industry's share in aggregate investment.

Rule 1 relates the creation of new capital stock in each industry to profitability in that industry. For a fuller explanation, the reader is referred to DPSV, Section 19. The effect of the equation is that industries which become more profitable attract more investment. The investment/capital ratio is related to the rate of return. It is to be interpreted as a risk-related relationship with relatively fast-(slow-) growing industries requiring premia (accepting discounts) on their rates of return.

Rule 2 may be used to determine investment in those industries for which Rule 1 is deemed inappropriate. These might be industries where investment is determined by government policy. For such an industry investment follows the national trend.

Rule 3 is intended for long run simulations, where the gross capital growth rates is fixed, with the effect that investment follows the industry capital stock.

Industry-specific investment follows one of three rules, according to the closure’ specification. In particular, rules 1 and 2 are designed for short-run simulations, contrarily to rule 3 which is a long-run rule. In the short run, we use rule 1 for those industries whose investment is related to the industry's profitability and rule 2 for those industries whose investment demand does not fit this relationship. Contrarily, in long run simulations rule 3 applies to all industries.

Finally, via an economy wide-rate of return, aggregate investment can be exogenised, or linked to aggregate real consumption. A last equation may be used to enforce the rule that capital is fixed in aggregate, yet freely mobile between sectors.

2.4.17 The labour market

The core ORANI-IT contains no theory of labour supply. Users of the model have the option of setting employment exogenously, with market-clearing wage rates determined endogenously, or setting wage rates (real or nominal) exogenously, allowing employment to be demand determined.

In the standard ORANI-IT short-run closure, all wages are indexed to the CPI. Then, shocking a shifter variable allows for deviations in the growth of some or all wages relative to the growth of the
CPI. As a variant on the standard short run closure, the average nominal wage may be fixed, with the effect of giving the model a Keynesian flavour.

The typical long-run closure sets the aggregate employment (wage bill weights) exogenous, with fixed wage relativities. This assumes that labour is mobile between industries and occupations.

These labour-market modelling decisions are usually made at an economy-wide level, but could be applied individually and perhaps differentially to different industries or types of labour. For example, we could exogenise the supply of skilled workers and the wages of unskilled workers. Or, we could exogenise employment in agriculture and wages in other sectors.

In the standard long-run and short-run closures, wage relativities are fixed, so firms do not substitute between labour of different types. Thus the values of the elasticity of substitution, the CES substitution elasticity between skill types, do not affect simulation results. This is handy for the modeller who is agnostic about those values (econometric evidence is scant). The more exotic labour market closures, which do allow wage relativities to change, focus attention on the values assumed for the elasticity of substitution.

2.5 A comparative-static interpretation of model results

ORAN-IT is static, with the time dimension, split into long and short run, implied by the variables’ allocation between the endogenous and exogenous categories.

The model is designed for comparative-static analysis. Once the model is closed, the economy is shocked according to the selected policy, and solved for the counterfactual equilibrium. Results of a simulation for a counterfactual equilibrium are expressed as percentage changes deviation from the benchmark economy, with no track of the evolution path of the variables. Therefore, the model is informative on the effect of a shock on the model’s variables, by comparing the economy with and with no policy, at a certain time t, which can be short or long.

This interpretation is illustrated by Figure 9, which graphs the values of some variable, say employment, against time. A is the level of employment in the base period (period 0) and B is the level which it would attain in T years time if some policy—say a tariff change—were not implemented. This is called the baseline, or business as usual forecast. With the tariff change, employment would reach C, all other things being equal. In a comparative-static simulation, ORANI-IT might generate the percentage change in employment 100(C-B)/B, showing how employment in period T would be affected by the tariff change alone.
ORANI-IT can be used to run simulations to analyse the short-run effects of policy changes. For these simulations, capital stocks are usually held at their pre-shock levels. Econometric evidence suggests that a short-run equilibrium will be reached in about two years, i.e., $T=2$ (Cooper, McLaren and Powell, 1985). ORANI-IT can also be used to assess long-run effects of policy change. In that case, it adopts the long-run assumption that capital stocks will have adjusted to restore (exogenous) rates of return—this might take 5 or 10 years, i.e., $T=5$ or 10. In either case, only the choice of closure and the interpretation of results bear on the timing of changes: the model itself is atemporal. Consequently it tells us nothing of adjustment paths, shown as dotted lines in Figure 9.

3 The model database

This section discusses the compilation of the database for the ORANI-IT model. The database consists of two main parts: input-output data of the base year, which provides the initial solution to the model; and behavioural parameters, which govern matters such as how economic agents respond to changes in relative prices.

3.1 Data source for the core database

CGE models attempt to capture all interdependencies arising from the constraints which bind the economy as a whole. Resource endowment and balance of payment are examples. Through these, demands of agents in each market affect all the economy via movements in factor prices, in the exchange rate, in costs of production and then in good prices. In order to capture all these interdependencies national accounts data are required. We use the Italian Supply-Use tables\(^{11}\) for 2008, structured into four tables:

- a USE table at purchasers prices for the combined domestic and imported varieties of each commodity;

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\(^{11}\) Downloaded at [http://www.istat.it.it/archivio/60913](http://www.istat.it.it/archivio/60913)
• a USE table at basic prices for the combined domestic and imported varieties of each commodity;
• an Imports table at basic prices;
• a Supply Table, consisting of a multi-production matrix, size 63 commodity by 63 industry, mainly but not wholly diagonal and 3 columns showing total imports, margins and net taxes for each commodity, used to reconcile basic and purchasers values.

3.2 Compilation of the ORANI-IT Input-Output Database

To convert the data sources described in the previous section to a database suitable by ORANI-IT (as shown in Figure 1), we have conducted the following procedures.

First, the values for V1BAS to V6BAS in row 1 are deduced from the USE table at basic prices and the Imports table at basic prices. Note, however, that at this stage the rows for margin commodities contain both direct use and margin use of these commodities by different users. These rows will be corrected when we estimate the margin uses of these commodities in the next step.

Second, we derive the tax and margin matrices in rows 2 and 3, using the following information from the SUPPLY and USE tables as constraints: the total value of commodity taxes on each commodity, summed over users, reported in column “Taxes” of the SUPPLY table; the total value of margins on each commodity, summed over users, reported in column “Margins” of the SUPPLY table; the sum of direct and margin use of each margin commodity by each user on all commodities used by the user, reported as the rows for margin commodities in the USE matrix at basic price; and the matrix (COM * USER) showing the sum of commodity taxes and margins for each commodity flow to each of the users. This matrix is derived by subtracting the USE matrix at basic prices from the USE matrix at purchaser’s prices. To derive the tax and margin matrices, we first allocate the total taxes and margins using pro rata assumptions. We then scale the matrices iteratively in order to meet the 4 constrains described above. Readers interested in this procedure can refer to Appendix E for more details.

Third, we take the “Compensation of employees” data from the USE matrix at basic prices to form the V1LAB matrix in Figure 1. Note that the USE matrix does not provide any data on occupational composition of employment in each industry. Therefore, the occupation dimension of V1LAB contains only one element representing all occupations.

Fourth, we deduce the VICAP and V1LND vectors (rows 5-6). Combining the “Net Operating Surplus” and “Depreciation” for each industry in the USE table at basic prices to form a single vector for gross return on capital, because the concept of capital rentals in ORANI-IT is gross, not net. The Italian Supply and Use tables do not provide data on land rentals. We assume that they are included in the capital rentals. Therefore, for land and resource using industries, such as agriculture, forestry, fishing and mining, we reallocate some of the capital rentals to land rentals based on an assumed land shares. At this stage, we borrow estimates for land shares from the GTAP database for Italy. This number can be revised when actual land rental data become available.

Finally, we take the data in the “Other taxes on production” in the USE matrix at basic price to form the V1PTX vector. The V1OCT vector is assumed to contain all zeros.

3.3 Parametrization

Apart from the national accounts data, in order to calibrate the model, elasticities and behavioural parameters are required to mimic how agents respond to changes in quantities and prices.
Choosing a numeraire, all benchmark equilibrium prices are thus 1 euro and the observed values are the benchmark quantities. The equilibrium conditions of the model are then used to determine the behavioural equation parameters consistent with the benchmark data set. In such a way, the model is calibrated, in the sense that the benchmark data can be reproduced as an equilibrium. Most of the values for elasticities are chosen from the review of the empirical work of IGEM (Annicchiarico, Di Dio, Felici, Monteforte, 2013) or from the GTAP database (Narayanan et al. 2012).

Starting with the producer theory, the primary factor nest requires an estimate of the substitution parameters between the primary factors, labour and capital. We borrow the econometric estimate of Edwin van der Werf (2008), which sets the value for Italy at 0.5. In the same study, the author also tests the hypothesis of a unity elasticity, rejecting the hypothesis of a Cobb-Douglas production function. This result supports our choice of a CES functional form\textsuperscript{12}.

Turning to the nests of demand for labour, estimates of substitution parameters between ages, genders, sectors and professional position are required. Considerable uncertainty exists in the literature. Following Annicchiarico, Di Dio, Felici, Monteforte (2013) we set the values at 0.35.

The CRETH product-product transformation parameters are set to 0.5 for multi-product industries. For single-product industries the estimates is not required, as the price transformation term disappears.

The short-run elasticity of real investment with respect to rates of return to capital is set at 0.1. In the long run, pressure on the capital market is absorbed by movements in the capital stock. Therefore, the level of investment is bounded by the investment to capital ratio.

ORANI-IT models the substitution possibility between domestic and foreign sources of supply via an Armington elasticity\textsuperscript{13}. Its value is set according to the GTAP estimates, and is assumed to be the same for all users (which is a reasonable assumption if the major imports are predominately in one end-use category only).

We then turn to the reciprocal of the export demand elasticity, which can be interpreted as the elasticity of Italy’s aggregate export volumes to the foreign currency price of Italy’s exports. Its value depends on the market power for individual export commodities that Italy can exert. Following estimates from GTAP, higher values of the elasticity correspond to the so called "made in Italy", which includes several goods, such as luxury goods (cars, fashion), food, and some high-skilled manufacturing products (chemicals, plastics). At the moment the value for collective exports is 4.

Household expenditure and price elasticities of demand derive from the GTAP database.

Given values for elasticities in the model, the equilibrium conditions can be calibrated to the benchmark data set.

\textsuperscript{12} Despite similarities in the estimated elasticities, predicted outcomes of the Cobb-Douglas or CES can be quite different because of interaction with key components of the CGE model, in particular the response of elasticities of factor to reward differentials, as discussed by Adkins, Rickmand and Hameed (2002)

\textsuperscript{13} Armington assumption treats imperfect substitution possibilities between imported and domestic varieties as being governed by a CES function. An elasticity greater than 1 implies that import and domestic varieties are relatively good substitutes, a value between 0 and 1 implies that are relatively poor substitutes
4 Features of ORANI-IT

For constructing the database, the first piece of information is represented by national accounts data. Then, depending on the nature of the model and on the desired level of detail, additional data sources can be required. However, when dealing with highly detailed models, data availability and/or format can come to represent a shortcoming. When this happens to be the case, to compensate for the lack of data, the use of assumptions is required.

Taking advantage of data availability, we replace some of the assumptions typically used in ORANI-style model with actual data, improving the reliability of model’s results. We also expand the modelling of labour demand, to allow for a deeper level of analysis on the labour market.

4.1 Investment matrix

ORANI includes an investment matrix, presenting gross fixed investment by commodity and industry.

Across the data sources used by ORANI-IT, the USE table with data on gross fixed investment by commodity allows for the construction of the commodity dimension. Investments by commodity so constructed are then broken-down across industries in proportion to their capital stock, with the underlined assumptions that all industries have the same commodity composition of investment expenditures and the same rate of return to capital.

Departing from this approach, we replace the adopted assumption with actual data on investment by 37 industry, released by ISTAT\(^{14}\). Note that in order to exploit the additional data source in the construction of a more reliable investment matrix, the data manipulation procedure requires an initial investment pattern, that we borrow from the Monash model (The MONASH model is fully documented in the book Dynamic General Equilibrium Modelling for Forecasting and Policy: A Practical Guide and Documentation of MONASH published by North-Holland in 2002).

The output is a better estimate of the investment matrix.

4.2 Demand of labour by industry, gender, age and professional position

ORANI present a specification of labour demand by industry. Taking advantage of data availability, we further specify the demand of labour via the inclusion of additional details. In particular, we collect micro-data on labour income from the IT-SILC database - provided by the Ministry of Economy and Finance – featuring a disaggregation by: gender, age, sectors, and professional position. The purpose is to enable a more informative assessment of the impact of shocks on the demand of labour, as well as of market-labour related policies, such as reforms of taxes on labour.

In the standard ORANI, demand of labour in each industry results from the solution to an optimisation problem, which minimizes the total cost of labour. As substitution possibilities are governed by choice of functional forms, we adopt a CES functional form to model the demand of labour for each of the characteristics. In practice, starting from the demand of labour by industry, we introduce additional nests, each of which describing the cost minimization problem faced by firms.

Specifically, we model the demand of labour in the following order. Starting with the bottom nest, we consider first the demand by gender, followed by the demand by age, classified in thirteen

\(^{14}\)Waterhouse database, downloaded at [http://dati.istat.it/Index.aspx?DataSetCode=DCCN_CAPITALTREV2](http://dati.istat.it/Index.aspx?DataSetCode=DCCN_CAPITALTREV2). Data on investment by 37 industry to the ORANI 63 industries were adapted by means of a mapping procedure, which relies on the ATECO classification of economic activity 2007
classes of age, and then the demand by professional position, classified as employees versus self-
employers, to conclude at the top nest with the existing demand by industry.

Solutions to the resulting minimization cost problems lead to key equations, which show the percentage change in the demand for alternative characteristics, within the same nest. These equations relate prices and quantities of different items in the same nest, taking into account the substitution effect between labour-types, caused by changes in their relative prices. Taking the professional position as an example, the ratio between the demand for employees (e) and for self-employers (s) is inversely related to their price ratio. The reasoning is summarized by the following formula:

\[ e = z - \sigma_{es}[p_e - (\alpha_s p_s + \alpha_e p_e)] \]  
\[ s = z - \sigma_{es}[p_s - (\alpha_s p_s + \alpha_e p_e)] \]  

By subtracting (20) - (21), we obtain:

\[ e - s = \sigma_{es}(p_e - p_s) \]  

We recall that all the equations in the model are written in percentage-change form. The percentage change in the demand for employees with respect to self-employees depends on the elasticity of substitution between the two professional positions \( \sigma_{es} \) and on the change in their relative prices. Changes in the relative prices within each characteristic induce substitution in favour of relatively cheaper characteristic.

5 Model's validation

Model's validation is a key issue when a new model is built. The purpose is to ensure a reliable and fruitful application of the model to policy analysis.

There are several levels of validation in CGE modelling. As we deal with a comparative-static model, following Dixon and Rimmer (2013) we consider those checks intended to make the model: computationally sound; built on up-to-dated and accurate data; capable of adequately capturing behavioural and institutional characteristics of the relevant part of the economy.

As first check, we demonstrate that the model has been computed correctly. With this aim, we look for coding and data-handling errors by means of homogeneity tests. In practice, first we run two simulations for which the correct results are known \textit{a priori}, as they exploit some of the model's theoretical features. Then, we test our model by comparing the generated results with the expected ones.

As the model is rooted in neoclassical theory, agents' decisions are only affected by relative prices, with no role for nominal prices. We run a nominal homogeneity test, by shocking the numeraire by a certain percentage change. In response to this, all endogenous nominal variables change by the same percentage, with all the real variables left unchanged. The test proves the model right, as no nominal rigidity has not been found.

We then run a real homogeneity test, which is set out on the model's property of constant returns to scale. As forecasted, a percentage increase in all real exogenous variables translate in a identical percentage increase in all of the real endogenous variables, with all the prices left unchanged.

The second step checks the initial data, by making sure that national accounts identities are satisfied. Checking initial data is relevant because the validity of percentage change equations depends on the validity of the data from which the equation coefficients are calculated. This check is set up around the fact that the SUT used for the construction of the model's database satisfy a
balance condition, according to which the sum of the basic values of activity outputs equals the sum of the basic values of demands for these outputs. It follows that the model is computationally sound if its database is balanced too.

In particular, we ensure that the model satisfies the following balancing conditions: the output of domestically produced commodities equals the total of the demands for them; the value of output by each industry equals the total of production costs. We then check that updated data-bases resulting from the simulations are balanced too.

The next step consists in validating the model through the GDP identity. We check the validity of CGE calculations, by ensuring that no discrepancy exists between GDP computed on the income side and on the expenditure side, in nominal and real terms. As explained in detail in Dixon and Rimmer (2013), this "provides a powerful device for checking the validity of CGE calculation...because the two measures of GDP involve distinct sets of variables which are linked indirectly through a large number of equations in the CGE framework" (p.1282).

Under the theoretical model's assumptions of zero pure profits and market clearing, the GDP identity continues to hold throughout all simulations, if the model has been properly implemented. Therefore, we ensure that the database is balanced and no errors in equations exist as changes in GDP resulting from the aforementioned simulations are the same from both sides.

Lastly, we verify the computational and theoretical validity of a set of CGE results, by conducting an exhaustive test simulation. Results analysis is an indispensable part of quality control for an economic model, as it reveals the model's properties, imperfections and weaknesses.

This kind of assessment can be qualitative or quantitative and is performed via the construction of a back-of-the-envelope (BOTE) model. The BOTE analysis allows to check if the model adequately captures behavioural and institutional characteristics of the economy, under the model's assumptions, by predicting how results depend on data, parameter values and behavioural assumptions. A detailed analysis of results can yield theoretical insights as it can happen that some minor mechanisms exert a dominant force in certain sectors, and can spotlight inappropriate theory. For the validation of ORANI-IT, we run an illustrative simulation, which is presented in the next section.

5.1 A test simulation: A consumption-neutral increase in national savings

5.1.1 Simulation design

The task of undertaking and interpreting simulations forms an important part of the checks on model implementation. In this section, we undertake a two-target/two-instrument simulation, which investigates a combined policy of a rise in the saving rates together with a lift to national productivity, that achieves a one-percentage-point of GDP increase in the balance of trade to GDP ratio, while leaving national real consumption unchanged.

In order to implement the simulation, first we use the model to find the magnitude of the joint change in the two instruments compatible with the aforementioned targets. Specifically, we run a simulation in which we shock the ratio of the balance of trade to GDP by +0.01, while holding real consumption unchanged. The model calculates that this would require a 1.38 per cent increase in primary factor productivity, and a 1.04 per cent fall in the ratio of consumption to GDP. Having found the changes in primary factor productivity and the savings rate, we run a second simulation to investigate the joint and separate long-run impacts of meeting the two policy targets on the economy at the macro and sectoral level.
In undertaking this simulation, our aim is illustrative, to test the model and demonstrate its capabilities. We do not contemplate or advocate specific policies or exogenous shocks that might create a rise in productivity or an increase in national savings. Nevertheless, if this were a live policy simulation, we conjecture that shocks of this type could easily be motivated in terms of the direct effects of a range of microeconomic reform policies (delivering a gain in productivity) and a range of policies to encourage private and/or public savings (delivering an increase in the national savings rate).

In the sections below we first discuss the closure of the model, and then the results of the simulations.

5.1.2 Model closure

We set the economic environment via our selection of exogenous/endogenous variables. As our aim is to understand the economic consequences of the shock after commodity and factor markets have had sufficient time to adjust, in undertaking the first simulation, we adopt a standard long-run closure, which is defined in the following terms.

Investors in each industry have had sufficient time to adjust capital stocks in response to the policy changes. Thus changes in demand for industry-specific capital are expressed as changes in capital supply, via an elastic supply to each industry at rates of return that are largely given. However, national aggregate capital stock is exogenous, with this accommodated via an endogenous economy-wide shifter on industry-specific rates of return. We assume that long-run national employment is exogenous, with the real national wage endogenous. We assume that the desired rate of capital accumulation in each industry in the long-run solution year is independent of the policy shock. We implement this via exogenous determination of industry investment/capital ratios. With movements in long-run industry-specific capital stocks largely determined by the first closure assumption above, this effectively links long-run movements in industry investment to movements in industry capital stocks. National investment is determined as the sum of industry investments. The ratio of the balance of trade to GDP is determined exogenously. With the solution year balance of trade constrained in this way, we allow private and public consumption to be determined endogenously via endogenous determination of the national savings rate.

5.1.3 Simulation Results

We examine the long-run consequences of joint increases in primary factor productivity and the national savings rate, achieved via a one percentage point rise in the ratio of the balance of trade to GDP; and, no change in real national consumption. To make clear the impacts of each of the shocks, we conduct a decomposition analysis based on the decomposition algorithm of Harrison et al. (2000). Tables 1 and 2 report the long-run impacts of the shock on the economy at the macro and sectoral level, distinguishing the joint effects (column 1) and the contribution of each of the two shocks (columns 2 and 3). Results in each of the columns 2 and 3 of the tables can be interpreted as the impact of the shock in the heading of the column in isolation of the shock in the other column.

Our discussion focuses on the joint impacts of both of the shocks, referring to the contribution of each of the shocks on changes in endogenous variables when appropriate.

5.1.3.1 Macroeconomic impacts

Table 1 reports the long-run macro impacts of the increases in both productivity and the saving rate. Rows 1 and 2 report the shocks: a 1.38 decrease in primary factor inputs per unit of output, and a 1.04 percent decline in the ratio of nominal consumption to nominal GDP. Column 1 reports the
total effect of the two shocks, and columns 2 and 4 isolate the individual effects of the productivity improvement and the rise in the savings rate.

We begin by considering the effects of the increase in the savings rate alone (column 3). The direct impact of the increase in the national saving rate is for private and public consumption to decline relative to what they would otherwise have been (rows 7 and 9, column 3). Under the model’s standard macro closure, consumption is assumed to move with GDP via a given propensity to consume. In column 3, the corollary of the rise in the savings rate is a fall in the propensity to consume. Hence, for a given level of real GDP (row 6), both private and public consumption decline (rows 7 and 9).

There is little scope in column 3 for real GDP to change. With employment and capital unchanged from what they would otherwise have been (rows 3 and 4, column 3), there is no change in real GDP at factor cost (row 5, column 3). With real GDP at factor cost unchanged, there is little scope for change in real GDP at market prices (row 6, column 3).

Real aggregate investment (row 8, column 3) is largely unaffected by the increase in the savings rate. This is because, under our long-run closure, industry-specific investment/capital ratios are exogenous. With the national capital stock unaffected by the shock (row 4, column 3) the scope for movement in aggregate investment is constrained by the fact that our closure specifies exogenous industry-specific investment/capital ratios.

With both real GDP and real investment largely unaffected by the increase in the savings rate, the declines in real private and public consumption cause real GNE to fall relative to real GDP. As such, the real balance of trade moves towards surplus (rows 10 and 11, column 3). This requires the real exchange rate to depreciate, rendering exports more attractive in foreign markets, and imports relatively more expensive in the domestic market (row 14, column 3). Expressed as a proportion of GDP, the movement towards surplus delivers approximately three quarters of the targeted movement in the balance of trade / GDP ratio (row 12, column 3).

We turn now to the effects of the change in productivity alone (column 2). With employment and capital exogenous and unshocked (rows 3 and 4, column 2), and the supply of agricultural land likewise held exogenous, the increase in primary factor productivity delivers a near identical increase in real GDP at factor cost (row 5). Just as we found when investigating column 3, in column 2 we also find little change in real investment (row 8, column 3), reflecting the fact that there is no change in the national capital stock (row 4, column 3).

In column 2, the ratio of nominal consumption to nominal GDP is unchanged. That is, in column 2 we hold the savings rate fixed. Hence, the increase in real GDP causes national consumption (rows 7 and 9) to rise relative to what it would otherwise have been. The percentage increase in real consumption is slightly lower than the percentage increase in real GDP because the terms of trade (row 15, column 2) decline relative to what they would otherwise have been. The decline in the terms of trade largely follows from the damped investment response. With the percentage increase in real investment being lower than the percentage increase in real GDP, the real balance of trade moves towards surplus (rows 10 and 11). The resulting expansion in export volumes causes the terms of trade to fall.

Column 2's increase in real net exports delivers the remaining quarter of the targeted movement towards surplus in the balance of trade / GDP ratio (row 12, column 2). Note also that, taken together, the productivity and savings shocks leave real private and public consumption unchanged (rows 7 and 9, column 1). This is by design: our savings and productivity shocks have been calibrated to jointly achieve this result. The savings shock alone (column 3) caused real private and public consumption to be 1.26 per cent lower than it would otherwise have been. This is neutralised
in column 2 by the productivity shock, which lifts real GDP by an amount sufficient to generate a 1.26 per cent increase in real national consumption.

Table 1. Macroeconomic impacts ( % )

<table>
<thead>
<tr>
<th>Variable</th>
<th>1.Total</th>
<th>2.Productivity</th>
<th>3.Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Ratio of consumption (private &amp; public) to GDP</td>
<td>-1.04</td>
<td>0.00</td>
<td>-1.04</td>
</tr>
<tr>
<td>2. Primary-factor-using technology</td>
<td>-1.38</td>
<td>-1.38</td>
<td>0.00</td>
</tr>
<tr>
<td>3. Employment</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4. Capital stock</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5. Real GDP (at factor cost)</td>
<td>1.39</td>
<td>1.39</td>
<td>0.00</td>
</tr>
<tr>
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5.1.3.2 Sectoral effects

Table 2 reports the sectoral effects of the shocks. ORANI-IT models the activity of 63 industries.

Table 2. Sectoral results

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The first thing we note is that, measured in terms of output expansion, all sectors, except equipment repairs and households own production, benefit (Column 1, Table 2). Some sectors experience relatively higher rates of output expansion. In part this reflects differences in the macroeconomic effects of the two components of the shocks, and in part it reflects sector-specific differences in costs and sales structures. These structures are reported in Tables 3 and 4 below. Table 3 reports the
destinations for the output of each sector. Table 4 reports the composition of each sector's production costs.
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Table 4. Sectoral cost structure (input costs as a percentage of total costs)
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In discussing the macroeconomic effects, we noted the strong movement towards balance of trade surplus (rows 10-12, column 1). In turning to the sectoral results, we might expect sectors with a strong trade focus to experience comparatively large expansions in output. Examining Table 3, we see that Mining and Manufactures have the highest export shares as a proportion of total sales. Turning to Table 2 (column 1), we find that, consistent with the pattern of macroeconomic outcomes discussed above, the sectors which benefit the most from the shocks are those with a strong trade focus: Mining and Manufactures. Manufactures experiences the largest output expansion, because it not only sells a relatively high proportion of its output to exports (Table 3), but it also has a high share of imported intermediate inputs in its production costs (Table 4). This sector gains from the expansion of export volumes and the depreciation of the exchange rate.

Construction and Services are the two sectors which experience the smallest output gains. These sectors sell relatively high proportions of their output to investment and consumption (Table 3). As discussed in the context of Table 1, neither the productivity improvement nor the rise in the savings rate has a large effect on economy-wide real investment. As such, the scope for output expansion is limited for Construction (Table 2), because it sells almost two-thirds of its output to investment-related activities (Table 3). For Services, its output expansion is comparatively damped because of the impact of the increase in the savings rate (column 3, Table 2). Just over half of all Services output is sold to private and public consumption (Table 3). As such, Services output is adversely affected by the increase in the savings rate in column 3.

5.1.4 Concluding remarks on simulation results

An important strength of large-scale CGE models is their industrial and commodity detail. This detail proves valuable in identifying winners and losers from policy change. In choosing an illustrative simulation for ORANI-IT, we were guided by a desire to highlight the model's capacity to identify winners and losers, and to inform policy design in ways that might mitigate policy adjustment costs. In our illustrative simulation, the increase in the national savings rate has diverse impacts on industries. Indeed, output in consumption-oriented industries is projected to be lower than it would otherwise be. However, paired with a program of productivity-enhancing microeconomic reform, these potential adjustment costs are mitigated. The results of the joint simulation indicate the potential for all industries to experience outputs gains. These gains are somewhat weighted more towards industries with greater trade exposure, although this follows naturally from a key element of the simulation design, namely, the targeting of a rise in the ratio of the balance of trade to GDP.

6 Conclusion

This paper presents a comparative-static national multi-sectoral computable general equilibrium model of Italy. The paper contributes to the existing Italian literature, as the model is designed at the Ministry of Economy and Finance, in collaboration with the Centre of Policy Studies, and is intended to provide practical policy recommendation. A static general equilibrium methodology for policy analysis is illustrated, with a detailed description of the model’s theoretical framework and database. The performance of an illustrative simulation has proven the model capable of assessing the economic impact of a desirable policy, at national and sectoral level. In particular, the multi-sectoral supply-demand interaction and the multi-production structure of the economy make the model especially well-suited for the assessment of industrial reforms. Future research may be pursued in the application of the model for policy analysis. In particular, new insights on the Italian industrial policy may be drawn by means of the developed model.
Appendix A. The percentage-change approach to model solution

A typical CGE model can be represented in the levels as:

\[ F(Y,X) = 0, \quad (A1) \]

where \( Y \) is a vector of endogenous variables, \( X \) is a vector of exogenous variables and \( F \) is a system of non-linear functions. The problem is to compute \( Y \), given \( X \). Normally we cannot write \( Y \) as an explicit function of \( X \).

Several techniques have been devised for computing \( Y \). The linearised approach starts by assuming that we already possess some solution to the system, \( \{Y^0,X^0\} \), i.e.,

\[ F(Y^0,X^0) = 0. \quad (A2) \]

Normally the initial solution \( \{Y^0,X^0\} \) is drawn from historical data—we assume that our equation system was true for some point in the past. With conventional assumptions about the form of the \( F \) function it will be true that for small changes \( dY \) and \( dX \):

\[ F_Y(Y,X)dY + F_X(Y,X)dX = 0, \quad (A3) \]

where \( F_Y \) and \( F_X \) are matrices of the derivatives of \( F \) with respect to \( Y \) and \( X \), evaluated at \( \{Y^0,X^0\} \).

For reasons explained below, we find it more convenient to express \( dY \) and \( dX \) as small percentage changes \( y \) and \( x \). Thus \( y \) and \( x \), some typical elements of \( y \) and \( x \), are given by:

\[ y = 100dY/Y \quad \text{and} \quad x = 100dX/X. \quad (A4) \]

Correspondingly, we define:

\[ G_Y(Y,X) = F_Y(Y,X)Y,^\wedge \quad \text{and} \quad G_X(Y,X) = F_X(Y,X)X,^\wedge, \quad (A5) \]

where \( Y,^\wedge \) and \( X,^\wedge \) are diagonal matrices. Hence the linearised system becomes:

\[ G_Y(Y,X)y + G_X(Y,X)x = 0. \quad (A6) \]

Such systems are easy for computers to solve, using standard techniques of linear algebra. But they are accurate only for small changes in \( Y \) and \( X \). Otherwise, linearisation error may occur. The error is illustrated by Figure 2, which shows how some endogenous variable \( Y \) changes as an exogenous variable \( X \) moves from \( X^0 \) to \( X^F \). The true, non-linear relation between \( X \) and \( Y \) is shown as a curve. The linear, or first-order, approximation:

\[ y = - G_Y(Y,X)^{-1}G_X(Y,X)x \quad (A7) \]

leads to the Johansen estimate \( Y^J \)—an approximation to the true answer, \( Y^{\text{exact}} \).
Figure 1 suggests that, the larger is $x$, the greater is the proportional error in $y$. This observation leads to the idea of breaking large changes in $X$ into a number of steps, as shown in Figure 2. For each sub-change in $X$, we use the linear approximation to derive the consequent sub-change in $Y$. Then, using the new values of $X$ and $Y$, we recompute the coefficient matrices $G_Y$ and $G_X$. The process is repeated for each step. If we use 3 steps (see Figure 2), the final value of $Y$, $Y^3$, is closer to $Y^{\text{exact}}$ than was the Johansen estimate $Y^J$. We can show, in fact, that given sensible restrictions on the derivatives of $F(Y,X)$, we can obtain a solution as accurate as we like by dividing the process into sufficiently many steps.

The technique illustrated in Figure 2, known as the Euler method, is the simplest of several related techniques of numerical integration—the process of using differential equations (change formulae) to move from one solution to another. GEMPACK offers the choice of several such techniques. Each requires the user to supply an initial solution $\{Y^0, X^0\}$, formulae for the derivative matrices $G_Y$ and $G_X$, and the total percentage change in the exogenous variables, $x$. The levels functional form, $F(Y,X)$, need not be specified, although it underlies $G_Y$ and $G_X$.

The accuracy of multistep solution techniques can be improved by extrapolation. Suppose the same experiment were repeated using 4-step, 8-step and 16-step Euler computations, yielding the following estimates for the total percentage change in some endogenous variable $Y$:

- $y(4\text{-step}) = 4.5\%$,
- $y(8\text{-step}) = 4.3\%$ (0.2\% less), and
- $y(16\text{-step}) = 4.2\%$ (0.1\% less).

Extrapolation suggests that the 32-step solution would be:

- $y(32\text{-step}) = 4.15\%$ (0.05\% less),

and that the exact solution would be:

- $y(\infty\text{-step}) = 4.1\%$.  

The extrapolated result requires 28 (= 4+8+16) steps to compute but would normally be more accurate than that given by a single 28-step computation. Alternatively, extrapolation enables us to obtain given accuracy with fewer steps. As we noted above, each step of a multi-step solution requires: computation from data of the percentage-change derivative matrices $G_Y$ and $G_X$; solution of the linear system (A6); and use of that solution to update the data (X,Y).

In practice, for typical CGE models, it is unnecessary, during a multistep computation, to record values for every element in X and Y. Instead, we can define a set of *data coefficients* V, which are functions of X and Y, i.e., $V = H(X,Y)$. Most elements of V are simple cost or expenditure flows such as appear in input-output tables. $G_Y$ and $G_X$ turn out to be simple functions of V; often indeed identical to elements of V. After each small change, V is updated using the formula $v = H_Y(X,Y)y + H_X(X,Y)x$. The advantages of storing V, rather than X and Y, are twofold:

- the expressions for $G_Y$ and $G_X$ in terms of V tend to be simple, often far simpler than the original F functions; and
- there are fewer elements in V than in X and Y (e.g., instead of storing prices and quantities separately, we store merely their products, the values of commodity or factor flows).

Levels and linearised systems compared: a small example

To illustrate the convenience of the linear approach\textsuperscript{15}, we consider a very small equation system: the CES input demand equations for a producer who makes output Z from N inputs $X_k$, $k=1-N$, with prices $P_k$. In the levels the equations are (see Appendix B):

$$X_k = Z \delta^{1/(\rho+1),k} \left[ \frac{P_k}{P_{ave}} \right]^{-1/(\rho+1),k}, \quad k=1,N$$  \hspace{1cm} (A8)

where $P_{ave} = \left( \sum_{i=1}^{N} \delta^{1/(\rho+1),i} P_{i}^{\rho/(\rho+1),i} \right)^{(\rho+1)/\rho}$.  \hspace{1cm} (A9)

The $\delta_k$ and $\rho$ are behavioural parameters. To solve the model in the levels, the values of the $\delta_k$ are normally found from historical flows data, $V_k = P_k X_k$, presumed consistent with the equation system and with some externally given value for $\rho$. This process is called calibration. To fix the $X_k$, it is usual to assign arbitrary values to the $P_k$, say 1. This merely sets convenient units for the $X_k$ (base-

\textsuperscript{15} For a comparison of the levels and linearised approaches to solving CGE models see Hertel, Horridge & Pearson (1992).
period-dollars-worth). \( \rho \) is normally given by econometric estimates of the elasticity of substitution, \( \sigma = (1/(\rho + 1)) \). With the \( P_k, X_k, Z \) and \( \rho \) known, the \( \delta_k \) can be deduced.

In the solution phase of the levels model, \( \delta_k \) and \( \rho \) are fixed at their calibrated values. The solution algorithm attempts to find \( P_k, X_k \) and \( Z \) consistent with the levels equations and with other exogenous restrictions. Typically this will involve repeated evaluation of both (A8) and (A9)—corresponding to \( F(Y, X) \)—and of derivatives which come from these equations—corresponding to \( F_Y \) and \( F_X \).

The percentage-change approach is far simpler. Corresponding to (A8) and (A9), the linearised equations are (see Appendices B and D):

\[
x_k = z - \sigma(p_k - p_{ave}), \quad k=1,N \tag{A10}
\]

and \( p_{ave} = \sum_{i=1}^{N} S_i p_i \), where the \( S_i \) are cost shares, eg, \( S_i = V_i / \sum_{k=1}^{N} V_k \) \tag{A11}

Since percentage changes have no units, the calibration phase—which amounts to an arbitrary choice of units—is not required. For the same reason the \( \delta_k \) parameters do not appear. However, the flows data \( V_k \) again form the starting point. After each change they are updated by:

\[
V_{k,\text{new}} = V_{k,\text{old}} + V_{k,\text{old}}(x_k + p_k)/100 \tag{A12}
\]

GEMPACK is designed to make the linear solution process as easy as possible. The user specifies the linear equations (A10) and (A11) and the update formulae (A12) in the TABLO language—which resembles algebraic notation. Then GEMPACK repeatedly:

- evaluates \( G_Y \) and \( G_X \) at given values of \( V \);
- solves the linear system to find \( y \), taking advantage of the sparsity of \( G_Y \) and \( G_X \); and
- updates the data coefficients \( V \).

The housekeeping details of multistep and extrapolated solutions are hidden from the user. Apart from its simplicity, the linearised approach has three further advantages.

- It allows free choice of which variables are to be exogenous or endogenous. Many levels algorithms do not allow this flexibility.
- To reduce CGE models to manageable size, it is often necessary to use model equations to substitute out matrix variables of large dimensions. In a linear system, we can always make any variable the subject of any equation in which it appears. Hence, substitution is a simple mechanical process. In fact, because GEMPACK performs this routine algebra for the user, the model can be specified in terms of its original behavioural equations, rather than in a reduced form. This reduces the potential for error and makes model equations easier to check.
- Perhaps most importantly, the linearized equations help us understand simulation results. We can easily see the contribution of (the change in) each RHS variable to the LHS of each equation. For example, in the CES price index equation:

\[
p_{ave} = \sum_{i=1}^{N} S_i p_i
\]

we can identify the contribution of each individual price \( p_i \) to the index \( p_{ave} \). The GEMPACK program AnalyseGE automates this task.
Appendix B: Percentage-Change Equations of a CES Nest

Problem: Choose inputs $X_i$ \((i = 1 \text{ to } N)\), to minimise the cost $\sum_i P_i X_i$ of producing given output $Z$, subject to the CES production function:

$$Z = \left(\sum_i \delta_i X_i^{-\rho}\right)^{-1/\rho}.$$  \hfill (B1)

The associated first order conditions are:

$$P_k = \Lambda \frac{\partial Z}{\partial X_k} = \Lambda \delta_k X_k^{-\rho} \left(\sum_i \delta_i X_i^{-\rho}\right)^{-1/\rho}$$  \hfill (B2)

Hence

$$\frac{P_k}{P_i} = \frac{\delta_k}{\delta_i} \left(\frac{X_k}{X_i}\right)^{1+\rho}$$  \hfill (B3)

or

$$X_i^{-\rho} = \left(\frac{\delta_i P_k}{\delta_k P_i}\right)^{\rho+1} X_k^{-\rho}.$$  \hfill (B4)

Substituting the above expression back into the production function we obtain:

$$Z = X_k \left(\sum_i \delta_i \left[\frac{\delta_k P_i}{\delta_i P_k}\right]^{\rho+1} -\rho\right)^{-1/\rho}.$$  \hfill (B5)

This gives the input demand functions:

$$X_k = Z \left(\sum_i \delta_i \left[\frac{\delta_k P_i}{\delta_i P_k}\right]^{\rho+1}\right)^{1/\rho},$$  \hfill (B6)

or

$$X_k = Z \delta^{1/(\rho+1)} \left[\frac{P_k}{P_{\text{ave}}}\right]^{1/\rho}.$$  \hfill (B7)

where

$$P_{\text{ave}} = \left(\sum_i \delta_i^{1/(\rho+1)} \frac{P_i^{\rho}}{P_{\text{ave}}^{\rho+1}}\right)^{(\rho+1)/\rho}.$$  \hfill (B8)

Transforming to percentage changes we get:

$$x_k = z - \sigma \left(P_k - P_{\text{ave}}\right),$$  \hfill (B9)

and

$$P_{\text{ave}} = \sum_i S_i P_i,$$  \hfill (B10)

where

$$\sigma = 1/\rho+1 \text{ and } S_i = \delta_i^{1/(\rho+1)} \frac{P_i^{\rho}}{P_{\text{ave}}^{\rho+1}} / \sum_k \delta_k^{1/(\rho+1)} P_k^{\rho}/(\rho+1)$$  \hfill (B11)

Multiplying both sides of (A7) by $P_k$ we get:

$$P_k X_k = Z \delta_k^{\rho+1} P_k^{\rho+1} P_{\text{ave}}^{\rho+1}$$  \hfill (B12)

Hence

$$\frac{P_k X_k}{\sum_i P_i X_i} \frac{1}{\sum_i \delta_i^{\rho+1} P_i^{\rho+1}} = S_k$$  \hfill (B13)

i.e., the $S_i$ of (B11) turn out to be cost shares.

**Technical Change Terms**

With technical change terms, we must choose inputs $X_i$ so as to:

$$\text{minimise } \sum_i P_i X_i \text{ subject to: } Z = \left(\sum_i \delta_i \left[X_i^{-\rho}\right]\right)^{-1/\rho}. $$  \hfill (B14)

Setting $\bar{X}_i = \frac{X_i}{A_i}$ and $\bar{P}_i = P_i A_i$ we get:

$$\text{(B15)}$$
minimise $\sum_i \bar{p}_i \bar{x}_i$ subject to: $Z = \left( \sum_i \delta_i \bar{x}_i^{-\rho} \right)^{-1/\rho}$, \hfill (B16)

which has the same form as problem (B1). Hence the percentage-change form of the demand equations is:

$$\bar{x}_k = z - \sigma(\bar{p}_k - \bar{p}_{ave})$$ \hfill (B17)

and $\bar{p}_{ave} = \sum_i S_i \bar{p}_i$ \hfill (B18)

But from (B15), $\bar{x}_k = x_i - a_i$, and $\bar{p}_i = p_i + a_i$, giving:

$$x_k - a_k = z - \sigma(p_k + a_k - \bar{p}_{ave})$$ \hfill (B19)

and $\bar{p}_{ave} = \sum_i S_i (p_i + a_i)$. \hfill (B20)

When technical change terms are included, we call $\bar{x}_k$, $\bar{p}_k$ and $\bar{p}_{ave}$ effective indices of input quantities and prices.

**Two Input CES: Reverse Shares**

Where a CES nest has only two inputs we can write (B19) and (B20) in a way which speeds up computation. Suppose we have domestic and imported inputs, with suffixes d and m. (B19) becomes:

$$x_d - a_d = z - \sigma(p_d + a_d - S_d(p_d + a_d) - S_m(p_m + a_m)),$$

and

$$x_m - a_m = z - \sigma(p_m + a_m - S_d(p_d + a_d) - S_m(p_m + a_m)).$$ \hfill (B21)

Simplifying, we get:

$$x_d - a_d = z - \sigma S_m ((p_d + a_d) - (p_m + a_m)),$$

and

$$x_m - a_m = z + \sigma S_d ((p_d + a_d) - (p_m + a_m)).$$ \hfill (B22)

In order for TABLO to substitute out x, we must express (A22) as a single vector equation:

$$x_k - a_k = z - \sigma R_k ((p_d + a_d) - (p_m + a_m)),$$

k = d, m \hfill (B23)

The $R_k$ are reverse shares, defined by:

$$R_d = S_m \quad \text{and} \quad R_m = R_d - 1 = S_m - 1 = -S_d$$ \hfill (B24)

(note that $R_d - R_m = 1$)

(A20) becomes:

$$\bar{p}_{ave} = \sum_i S_i (p_i + a_i) = R_d(p_m + a_m) - R_m(p_d + a_d).$$ \hfill (B25)

**Twist for Two Input CES**

A twist is a combination of small technical changes which, taken together, are locally cost neutral. For example, we might ask, what values for $a_d$ and $a_m$ would, in the absence of price changes, cause the ratio $(x_d - x_m)$ to increase by $t\%$ without affecting $\bar{p}_{ave}$? That is, find $a_d$ and $a_m$ such that:

$$S_d a_d + S_m a_m = 0,$$ \hfill (B26)

using (A20), and

$$x_d - x_m = (1-\sigma)(a_d - a_m) = t,$$ \hfill (B27)

using (A21); giving
\[ a_d = S_m t/(1-\sigma) \quad \text{and} \quad a_m = -S_d t/(1-\sigma). \]  
(B28)

Adopting reverse share notation:
\[ a_k = R_k t/(1-\sigma) \quad k = d, m \]  
(B29)

Substituting (A29) back into (A23) we get:
\[ x_k = z + R_k t/(1-\sigma) - \sigma R_k (p_d - p_m + R_d t/(1-\sigma) - R_m t/(1-\sigma)) \quad k = d, m \]
so
\[ x_k = z + R_k t - \sigma R_k (p_d - p_m) \quad k = d, m \]
allowing us to rewrite (B19) and (A20) as:
\[ x_k = z + R_k (t - \sigma(p_d - p_m)) \quad k = d, m \]  
(B30)

and \( \tilde{p}_{ave} = R_d p_m - R_m p_d. \)  
(B31)

Twist variables, such as \( t, \) are heavily used in MONASH where they are used to simulate secular (i.e., not price-induced) trends in import shares. Figure A1 shows how 'twist' variables get their name. AA is an isoquant showing what quantities of domestic and imported goods can be combined to give the same utility. The chosen combination is at X. Technical changes \( a_d \) and \( a_m \) translate AA both down and to the right, in such a way that BB still passes through X. It is as if AA had been twisted or pivoted around X.

A small change concept of cost neutrality is used to develop the notion of twist variables. Where budget shares change by a large amount, the same technical change cannot be cost-neutral at both initial and final input proportions, although it will usually be cost-neutral at some intermediate proportion. Thus, there are no levels formulae corresponding to (B26).

**Appendix C: Algebra for the Linear Expenditure System**

The purpose of this appendix is to expand on some of the algebra underlying Excerpt 16 of the TABLO input file. First note that, the utility function (12) in the text can be written:

\[ \text{Utility per household} = \prod_c \left\{ \frac{x_3\ S(c)}{Q} - A3SUB(c) \right\}^{S3LUX(c)}, \]  
(C1)

using equation (17). Here, \( X_3\_S(c)/Q \) is the average consumption of each composite good \( c, \) and \( A3SUB(c) \) is a parameter. The household's problem is to choose \( X_3\_S(c)/Q \) to maximise utility subject to the constraint:

\[ \sum_c X_3\_S(c)/Q \cdot P3\_S(c) = V3TOT/Q, \]  
(C2)

The associated additional first order conditions are:
\[ \Delta P3\_S(c) = \frac{\partial u}{\partial (X_3\_S(c)/Q)} = S3LUX(c).U.\left\{ \frac{x_3\ S(c)}{Q} - A3SUB(c) \right\}^{-1} \]  
(C3)
Manipulation (and use of equation (17)) yields:
\[ P_{3_S}(c)(X_{3_S}(c) - X_{3SUB}(c)) = S3LUX(c).Q.U/\Lambda. \]  
(C4)

or  
\[ P_{3_S}(c).X_{3_S}(c) = P_{3_S}(c).X_{3SUB}(c) + S3LUX(c).Q.U/\Lambda. \]  
(C5)

(C5) is really the same as equations (15) to (17) in the text. The key to the simplicity of the equations there is that no attempt is made to eliminate the Lagrange multiplier term, Q.U/\Lambda. Instead, the constraint (C2) is written down as part of the equation system: see (18). This implicit approach often yields dividends. Here in the appendix we press forward more conventionally to express demands directly as a function of prices and income. Summing over c and using (C2), we see that
\[ Q*U/\Lambda = V3TOT - \sum_{c} P_{3_S}(c)*X_{3SUB}(c) \]  
(C6)

so that Q.U/\Lambda is identified as the V3LUX.C of equation (14) in the text. Equation (13) then follows from (C4) above. Combining (C4) with (C6) we get the linear expenditure system (LES):
\[ P_{3_S}(c)*X_{3_S}(c) = P_{3_S}(c)*X_{3SUB}(c) + S3LUX(c)(V3TOT - \sum_{k} X_{3SUB}(k)*P_{3}(k)). \]  
(C7)

To find the expenditure elasticities, we convert to percentage change form, ignoring all changes in prices and tastes:
\[ x_{3_s}(c) = - FRISCH.B3LUX(c)*w3tot \]  
(C8)

where FRISCH is defined, by tradition, as \[ \frac{V3TOT}{V3LUX.C} \], and the B3LUX(c) are the shares of 'luxury' in total expenditure on good c, i.e. \[ B3LUX(c) = \frac{X_{3_S}(c) - X_{3SUB}(c)}{X_{3_S}(c)} \].

Thus the expenditure elasticities are given by:
\[ EPS(c) = - FRISCH.B3LUX(c) \]  
(C9)

In the TABLO program, (C9) is reversed to derive B3LUX(c) from EPS(c) and FRISCH.

**Taste Change Terms**

Often we wish to simulate the effect of a switch in consumer spending, induced by a taste change. This could be be brought about by a shock either to the a3lux(c) (marginal budget shares) or to the a3sub(c) (subsistence quantities). Two problems arise. First, what combination of a3sub and a3lux shocks is best? Second, the a3lux shocks must obey the rule that marginal budget shares add to 1. To tie down the relation between the a3lux and the a3sub, we will assume that they move in proportion:
\[ a3lux(c) = a3sub(c) - \lambda, \]  
(C10)

and that the constant of proportionality \( \lambda \) is given by the adding-up requirement:
\[ \sum_{k} S3LUX(k).a3lux(k) = 0 \]  
(C11)

implying that:
\[ a3lux(c) = a3sub(c) - \sum_{k} S3LUX(k).a3sub(k) \]  
E_a3lux

We also suppose that
\[ \sum_k S_3 S(k).a_{3\text{sub}}(k) = 0 \quad \text{(C12)} \]

This is guaranteed by equation E_a3sub:

\[ a_{3\text{sub}}(c) = a_{3\text{s}}(c) - \sum_k S_3 S(k).a_{3\text{s}}(k) \quad \text{E_a3sub} \]

The effect of these assumptions is to allow budget shares to be shocked whilst altering expenditure elasticities and the Frisch parameter as little as possible.

**Appendix D: Short-Run Supply Elasticity**

Where capital stocks are fixed, we can derive an approximate expression for a short-run supply schedule as follows. Imagine that output, \( z \), is a CES function of capital and labour, and that other, material, inputs are demanded in proportion to output. Using percentage change form, we may write:

\[ p = H_K p_K + H_L p_L + H_M p_M \quad \text{zero pure profits} \quad \text{(D1)} \]

\[ x_L - x_K = \sigma(p_K - p_L) \quad \text{factor proportions} \quad \text{(D2)} \]

\[ z = S_L x_L + S_K x_K \quad \text{production function} \quad \text{(D3)} \]

where \( H_K, H_L \) and \( H_M \) are the shares in total costs of capital, labour and materials, and where \( S_K \) and \( S_L \) are the shares in primary factor costs of capital and labour. \( p \) is output price, and \( p_K, p_L \) and \( p_M \) are the prices of capital, labour and materials respectively. \( z \) is output, and \( x_K \) and \( x_L \) are the input quantities of capital and labour. "Capital" should be interpreted as covering all fixed factors, including land. In the short run closure we set \( x_K \) to zero, so that the last 2 equations become:

\[ x_L = \sigma(p_K - p_L) \quad \text{and} \quad z = S_L x_L \quad \text{giving:} \quad \text{(D4)} \]

\[ z = S_L \sigma(p_K - p_L) \quad \text{or} \quad p_K = p_L + z/(S_L \sigma) \quad \text{(D5)} \]

Substituting (J5) into (J1) we get:

\[ p = H_K (p_L + z/(S_L \sigma)) + H_L p_L + H_M p_M \quad \text{(D6)} \]

\[ p = z H_K/(S_L \sigma) + (H_K + H_L)p_L + H_M p_M \quad \text{(D7)} \]

\[ z H_K/(S_L \sigma) = p - (H_K + H_L)p_L - H_M p_M \quad \text{(D8)} \]

\[ z = \left( S_L \sigma/H_K \right)[p - (H_K + H_L)p_L - H_M p_M] \quad \text{(D9)} \]

Call \( H_F \) the share of primary factor in total costs (= \( H_K + H_L \)). Then \( H_K = S_K H_F \)

so \[ z = \left( \sigma S_L/S_K \right)[p/H_F - p_L - (H_M/H_F)p_M] \ldots \text{compare DPSV eq. 45.19} \quad \text{(D10)} \]

The short-run supply elasticity is the coefficient on \( p \), namely:

\[ \sigma S_L/(S_K H_F) \quad \text{(D11)} \]

In other words, supply is more elastic as either the labour/capital ratio is higher, or the share of materials in total cost is higher. (D11) is only a partial equilibrium estimate; it assumes that all inputs except capital are in elastic supply.

For industries which have permanently fixed factors (eg, land or natural resources) we could compute long-run supply elasticities in a similar way. In (D11) \( S_L \) should be interpreted as the share of *mobile* factors in primary factor cost, with \( S_K = 1 - S_L \).
Appendix E: Margin and Tax Matrices

In order to compute the margin and tax matrices we use a procedure which combines data information, assumptions and adjustment procedures together. Our procedure follows that of Horridge 2003.

Data on margins and taxes are contained in the Supply table, in the “trade and transport margins” and “taxes less subsidies on products” columns respectively. In particular, the margins column indicates the margin use only of such services, contrarily to the rows for margins services in the USE table at basic price which contain the sum of their margin and direct use. Using this information, the direct use of margins is computed as difference, under the assumption of no margin use on directly-used margin commodities. Therefore, as first step of our procedure, values for margins commodities in the USE table at basic price are split up in margin and direct use.

Coefficient

\[
\begin{align*}
&\text{(all,c,COM)(all,u,USR)} & \text{BAS(c,u)} & \text{Basic price USE, margins in margin rows} \\
&\text{(all,c,COM)(all,u,USR)} & \text{PUR(c,u)} & \text{Purchasers price USE} \\
&\text{(all,c,COM)(all,u,USR)} & \text{DIFF(c,u)} = \text{PUR(c,u)} - \text{BAS(c,u)} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{MARMAR(c,u)} & \text{Margin use on non-margin goods by user} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{DIRMAR(c,u)} & \text{Direct use of margin goods by user} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{TAXMAR(c,u)} & \text{Tax on directly-used margin goods by user} \\
&\text{(all,m,MAR)} & \text{SUPPMARG(m)} & \text{Margins in the Supply} \\
&\text{(all,m,MAR)} & \text{SUPPTAX(m)} & \text{Taxes less subsidies on products in the Supply} \\
&\text{(all,c,COM)} & \text{TOTDIRUSE(c)} & \text{Basic price Use, margin direct use only}
\end{align*}
\]

For Margin rows of the basic (BAS) and purchasers (PUR) price matrices:

\[
\begin{align*}
&\text{(all,c,MAR)(all,u,USR)} & \text{BAS(c,u)} = \text{DIRMAR(c,u)} + \text{MARMAR(c,u)} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{PUR(c,u)} = \text{DIRMAR(c,u)} + \text{TAXMAR(c,u)} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{DIFF(c,u)} = \text{PUR(c,u)} - \text{BAS(c,u)} = \text{TAXMAR(c,u)} - \text{MARMAR(c,u)} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{TAXMAR(c,u)} = \text{DIFF(c,u)} + \text{MARMAR(c,u)} \\
&\text{(all,c,MAR)(all,u,USR)} & \text{MARMAR(c,u)} = \text{TAXMAR(c,u)} - \text{DIFF(c,u)}
\end{align*}
\]

We then decompose the difference between USE at purchaser’s prices and USE at basic prices into margins and taxes. Our data disaggregation relies on the following five conditions, which must be satisfied:

\[
\begin{align*}
&1) \text{BAS(c,u)} = \text{DIRMAR(c,u)} + \text{MARMAR(c,u)} \\
&2) \text{PUR(c,u)} = \text{DIRMAR(c,u)} + \text{TAXMAR(c,u)} \\
&3) \sum\{u,USR, \text{MARMAR(m,u)}\} = - \text{SUPPMARG(m)} \\
&4) \sum\{u,USR, \text{TAXMAR(m,u)}\} = \text{SUPPTAX(m)} \\
&5) \sum\{u,USR, \text{DIRMAR(m,u)}\} = \text{DMAR(m)}
\end{align*}
\]

By using

\[
\text{PUR(c,u)} = \text{DIRMAR(c,u)} + \text{TAXMAR(c,u)}
\]

and assuming that

\[
\text{TAXMAR(c,u)} = \text{AVETAXRATE(c)*DIRMAR(c,u)},
\]

with the coefficient average tax rate on direct use

\[
\text{AVETAXRATE(c)} = \text{SUPPTAX(c) / TOTDIRUSE(c)}
\]

Which means that user specific commodity taxes are based on the assumption that all intermediate usage of a product is taxes at the same rate.
(all,c,MAR)(all,u,USR) \( \text{PUR}(c,u) = [1+\text{AVETAXRATE}(c)] \times \text{DIRMAR}(c,u) \)

(\text{all,c,MAR})(\text{all,u,USR}) \quad \text{DIRMAR}(c,u) = \frac{\text{PUR}(c,u)}{[1+\text{AVETAXRATE}(c)]}

(\text{all,c,MAR})(\text{all,u,USR}) \quad \text{TAXMAR}(c,u) = \text{PUR}(c,u) - \text{DIRMAR}(c,u)

(\text{all,c,MAR})(\text{all,u,USR}) \quad \text{MARMAR}(c,u) = \text{BAS}(c,u) - \text{DIRMAR}(c,u)

At this point, MARMAR, DIRMAR and TAXMAR are correctly signed and satisfied. Enforcing C, D and E means disturbing A and B, so an iterative scaling is needed:

A: Scale MARMAR, DIRMAR to add to BAS

B: Scale TAXMAR, DIRMAR to add to PUR

C: Scale MARMAR to add to (negative of) SUPPMARG Supply matrix column

D: Scale TAXMAR to add to SUPPTAX Supply matrix column

E: Scale DIRMAR to add to DMAR Supply matrix column
References


Ministry of Economy and Finance

Department of the Treasury

Directorate I: Economic and Financial Analysis

Address:
Via XX Settembre, 97
00187 - Rome

Websites:
www.mef.gov.it
www.dt.tesoro.it

e-mail:
dt.segreteria.direzione1@tesoro.it

Telephone:
+39 06 47614202
+39 06 47614197

Fax:
+39 06 47821886