

# On the design of Sovereign Bond-Backed Securities

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## Abstract

We analyze Sovereign Bond-Backed Securities, concentrating our attention on the return of the different tranches and on their risk. We show that as the correlation level among defaults increases, the yield rate of senior tranches increases while the yield rate of junior tranches decreases. A similar effect is observed when introducing a block dependence structure with high correlation among states belonging to the same block. Introducing a non-zero recovery rate, as opposed to a null recovery rate, decreases the yield rate of senior tranches and increases the yield rate of junior tranches. We compute the loss distribution and the Value at Risk (VaR) associated with the market risk of detaining the different tranches of the bond. We also analyze the possibility of reaching a safe asset through pooling tranches of government bonds of different States. In summary, we show that the issue in reaching a comprehensive and safe offering through the securitization of government bonds is not the safety of senior tranches but the risk of the junior ones.

**Keywords:** Sovereign bond-backed securities, tranche, diversification, Euro.

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# 1 Introduction

The Euro crisis shook the European Monetary Union to its roots. At source, the issue is the one identified in [11]: in a monetary union, national governments issue debt in a currency that is not their own. In the European environment, the key issue is provided by the presence of the no bail-out clause contained in the Maastricht Treaty (art.123 and 125): the responsibility for repaying public debt remains at the national level, ruling out the possibility of interventions from other States or from the European Union. Moreover, the European Central Bank (ECB) cannot finance budget deficits by printing money.

Among the interventions to cope with the legacy of the financial crisis, the European Systemic Risk Board (ESRB) promoted a high level task force on safe assets. The task force has advanced the proposal to build Sovereign Bond-Backed Securities (SBBS), see [5, 15, 16, 22]. The European Commission has supported this attempt by issuing a regulation proposal to enable the development of SBBS, see [17]. Senior tranches of SBBS would represent a low-risk asset characterized by high liquidity, low volatility and low credit risk in normal and crisis periods. In this paper we analyze the design of SBBS concentrating our attention on the yield rate of the different tranches and on their riskiness. We show that senior tranches are safe and their yield is small and stable across many modeling assumptions. On the other hand we show that the yield of junior tranches is much more susceptible to modeling assumptions regarding the correlation level among default events and the recovery rate of sovereign bonds underlying the SBBS.

The Euro crisis was ignited by the financial crisis which led to huge recapitalizations of banks by States and called for expansive fiscal policies to tackle the crisis of the European economies. The high level of public debt/GDP ratio of some countries (the so-called non core countries) contributed to the crisis because the States did not have the weapons to bail-out banks and to implement expansive policies to stimulate the economy.

Three main strictly related issues emerged. a) The nexus between countries characterized by a high public debt and local banks detaining a large amount of national government bonds (home bias). This link played a destabilizing role during the Euro crisis with a self-reinforcing negative spiral: on one hand, banks were exposed to poor credit quality

government bonds and, on the other hand, (weak) States were not able to bail-out banks with taxpayers' money. These features undermined the balance sheet of banks and contributed to the credit crunch with negative consequences for the economy. b) Suddenly, the European money market fragmented, preventing a fluent transmission of the monetary policy and inducing excess volatility in the Euro-area, in particular the high level of public debt of some countries led to a flight to quality phenomenon inside the Euro area that exacerbated the financial turbulence (sale of poor credit quality government bonds and investment in high quality bonds). c) The crisis showed that credit and financial intermediaries need a safe asset as an anchor to be used as a collateral, to prevent issues resulting from the shortage of a safe asset (see [6, 28]). Because of the architecture of the European Union and of the Euro, a safe asset does not exist in the Euro area as government bonds are exposed to default risk and the stock of safest government bonds (e.g. German bonds) is not enough for the European banking system and, therefore, banks are short of collateral to cope with their funding needs. This point is strictly related to the fact that there is not a deep and liquid market for low risk government bonds in the Euro area and a sound reference interest rate curve is missing.

According to the intention of the ESRB, SBBS would address some of issues encountered during the Euro crisis. The existence of a safe asset obtained as a senior tranche of SBBS could help to weaken the bank-sovereign vicious circle (see [4]): banks could hold senior tranches of SBBS as safe asset instead of government bonds and this would help them to hold a diversified portfolio of government bonds reducing the home bias phenomenon.

It should be noticed that to reach both goals (diversification and low risk), both pooling of government bonds and seniority are needed, an asset backed by a diversified portfolio of sovereign bonds would not guarantee a risk reduction for more than half of the significant banks in the Euro area, assuming that they hold a fully diversified portfolio of government bonds instead of their actual portfolio. Instead, if banks disinvest their government bonds portfolio and re-invest some fraction of their current holdings in senior tranches of SBBS, they would get a significant de-risking, see [15,16]. A safe asset could also facilitate the integration of the money market, the transmission of the monetary policy and the funding of banks in difficult times; it could also limit the flight to quality phenomenon and could provide a benchmark for the Euro-yield rate curve.

SBBS have been deeply criticized on many fronts. Although these instruments could be issued by private intermediaries and they do not hinge on the joint liability by national States such as the Eurobonds proposed by [13], [1, 19] observed that they may be a source of moral hazard and redistribution of national risk from weak countries to solid countries. The root cause of moral hazard effects is provided by the need to bail-out junior tranches in case the market does not buy them anymore: states may be forced to intervene (buying junior tranches) to save the SBBS system. A second issue concerns the effect that the introduction of this asset would have on the market for sovereign bonds: SBBS should contribute to eliminate the mispricing and the turbulence due to the weak architecture of the Euro area but they should not interfere with the market evaluation of credit risk of the countries. Doubts also emerge on the possibility that senior tranches of SBBS will be considered safe during a crisis and will not be exposed to a flight to quality phenomenon, see [12]: the instrument may be considered a safe asset in normal time but not in crisis periods because in that case the correlation structure among countries would change dramatically to a strong correlation and investors may perceive SBBS as risky, see [12, 19, 21].

The aim of this paper is to test the robustness of the SBBS proposal. We concentrate on yield rates of different tranches and on their risk features. We show the following results taking the architecture of SBBS outlined in [15, 16] as a benchmark:

- Considering debt weights rather than GDP weights would lead to higher yield rates for all the tranches, in particular mezzanine and junior tranches.
- A positive recovery rate decreases the yield rate of senior tranches and increases the yield rate of junior tranches.
- Increasing the correlation level among State defaults increases the yield rate of senior tranches and decreases the yield rate of junior tranches. Vice versa a lower correlation leads to lower yields for the senior tranches and higher yields for junior tranches.
- Considering a block correlation structure (high correlation inside the block and weak correlation outside the block) increases the yield rate of senior tranches and decreases the yield rate of junior tranches. The results are confirmed considering

a two-block correlation structure.

From a quantitative point of view, the analysis shows that the correlation structure is not a big issue to obtain a safe asset and for the riskiness of junior bonds. Instead, the assumption of a zero recovery rate made in [15,16] leads to a significant underestimation of the yield rate of junior bonds. Considering a positive recovery rate and varying the correlation structure we observe that the two effects do not compensate each other: the yield rate of the safe tranche is low (below 20 basis points) but the yield rate of the junior tranche is quite high (above 500 basis points). Results are confirmed by varying the copula used to model correlation among defaults of States.

We also consider a different way to build a safe bonds as proposed in [23,24]: pooling senior tranches of government bonds issued by States. It turns out that a safe asset can be obtained at the cost of a high risk of junior tranches issued by States. These results show that the weakness in pooling bonds issued by different States at the Euro level is provided by the risk entailed in junior bonds.

The paper is organized as follows. In Section 2 we briefly describe SBBS. In Section 3 we analyze the robustness of SBBS investigating how the remuneration of the tranches depends on the weights of SBBS, on the recovery rate assumption and on the correlation structure. In Section 4 we investigate the robustness of SBBS with respect to the copula model adopted to establish correlation among defaults. In Section 5 we analyze a different hypothesis with tranching of national bonds followed by pooling. In Section 6 we analyze the riskiness of the tranches of SBBS. Section 7 concludes the paper.

## **2 Sovereign Bond-Backed Securities**

SBBS is a standard asset-backed security exploiting the two classical securitization principles: diversification and tranching. SBBS are built from a pool of sovereign bonds issued by EU member States (not necessarily adopting the Euro); bonds issued by local authorities and in other currencies are not allowed to be part of the pool. Bonds are purchased by the (public or private) intermediary/vehicle on the market at market prices. A market price transaction should avoid the possibility that the vehicle is used to rescue States in troubles buying their bonds at unrealistic prices. A market price transaction should also facilitate the evaluation of SBBS by investors. The weights of sovereign

bonds are provided by the capital key of the European Central Bank (ECB), which has been adopted as a benchmark in recent years under the Eurosystem's public sector purchase program. To maintain a liquid market for sovereign bonds, the total issuance of SBBS would be limited to at most 33% of total outstanding stock of sovereign bonds.

SBBS are designed to render senior tranches almost risk-free, see [14–16] . A tranching 70-20-10 would produce a senior tranche (70% of the pool) as safe as the German bund. The 20% layer defines the mezzanine tranche, the 10% layer defines the junior tranche. According to model simulations, the Expected Shortfall in the worst 1% scenarios would be 9% while that of German bonds is 13% (in the adverse scenario, these figures would be 26% and 37%, respectively); the yield rate of the senior tranche of SBBS on October 2016 would have been 0.13%, while on the same date the yield of German sovereign bonds was 0.08%, see [16]. No single sovereign default would trigger the 30% threshold with a loss on senior tranches. Under the assumption that multiple defaults of countries occur in ascending order of their credit rating and assuming a 100% loss given default (LGD), senior tranches would suffer a loss in case of defaults of Greece, Cyprus, Portugal, Italy and Spain; in case of a LGD equal to 40%, even the default of Germany would be needed for the senior tranche to suffer a loss. Notice that the average observed loss rate in sovereign debt restructuring historically is 37%, see [10]. Considering this reference value for the LGD, the five year expected shortfall of senior tranches of SBBS would be 0.5%. Riskiness of mezzanine tranches would be similar to that of lower investment grade Euro area sovereign bonds. The expected/unexpected loss of junior tranches would be similar to higher risk Euro area sovereign bonds or even above. The market size of SBBS could be around 1.5 trillion Euro.

The analysis developed in [15, 16] builds on quite standard assumptions. In this section we reproduce, in a different simulation framework, their results, providing us with a benchmark for our analysis. We concentrate our analysis on the yield of the tranches of SBBS built on a portfolio of 10 years (10Y) zero coupon bonds.

As in [15, 16], we consider eleven countries:

Austria, Belgium, Finland, France, Germany, Greece,  
Ireland, Italy, The Netherlands, Portugal, Spain.

For our computations, unless otherwise specified, we use the zero-coupon 10 years

Country	Weight	Country	Weight
Austria	3.21%	Ireland	2.06%
Belgium	3.87%	Italy	16.22%
Finland	2.00%	The Netherlands	6.63%
France	20.88%	Portugal	1.77%
Germany	28.08%	Spain	10.73%
Greece	1.98%		

Table 1: Average GDP over 2008-2017 as fraction of the EU area GDP.

(10Y) yield rates provided by Refinitiv for January 31, 2019. As in [15, 16], we choose the 10Y risk-free discount rate as the one of the less risky State, i.e., the one with lowest yield computed from the 10Y bonds (German bonds). The weights of the bonds included in the SBBS portfolio are taken from the country average GDP during the period 2008-2017. In Table 1 we report the average GDP data per country as fraction of the total euro area GDP. Notice that the sum of the weights is 97.43% due to the fact that we only include eleven countries in the pool of bonds. The weights of the zero coupon bond portfolio underlying the SBBS are those reported in Table 1 and they have been normalised to sum up to one.

To compute the yield rates of the different SBBS tranches we proceed as follows. Starting from the zero coupon bond data, we compute the 10Y default probability of each country. Then we simulate the default of each country in a 10 years horizon, correlating the single countries via a static Gaussian copula (see [27, Section 10.8] or [3, Chapter 3]). In this model the dependence structure is controlled by a correlation matrix that expresses how the defaults of different countries are related to each other. As baseline we assume a 60% flat default correlation, as done in [15, 16]. Once we have simulated one million scenarios, then we compute the loss on the zero coupon bond portfolio built renormalising the weights of Table 1 and having a notional of 1€. Then, for each scenario, we compute the losses of each tranche from the portfolio loss.

For example, let us consider a tranche structure of 70-20-10. In this framework, the junior tranche pays  $\frac{10-L_P}{10}$  with a zero-floor, where  $L_P$  is the portfolio loss expressed as a percentage of the notional portfolio. The mezzanine tranche pays the minimum between

Seniority	Senior	Mezzanine	Junior	Whole pool
70-20-10	0.27	1.94	4.12	0.92
80-10-10	0.38	2.76	4.12	0.92
70-0-30	0.27	-	2.62	0.92
80-0-20	0.38	-	3.43	0.92
90-0-10	0.62	-	4.11	0.92

Table 2: Yield rates of different tranches assuming GDP weights and no recovery rate. All data are in %, i.e., 0.27 corresponds to 0.27%. 70-20-10 means that the senior tranche represents 70% of the asset pool. By ‘Whole pool’ we denote the SBBS containing all the zero coupon bonds, without any tranching.

1 and  $\frac{30-LP}{20}$  with a zero-floor and the senior tranche pays the minimum between 1 and  $\frac{100-LP}{70}$  with a zero-floor. Hence, if the portfolio loses more than 10% of its value due to defaults then the junior tranche will be wiped out and the mezzanine tranche will suffer a loss; if the portfolio loses more than 30% of its value then both the junior and the mezzanine tranches will be wiped out and the senior tranche will suffer a loss. Finally, we compute the yearly yields of the different tranches as

$$\left(\frac{1}{P}\right)^{\frac{1}{10}} - 1,$$

where  $P$  is the current price of the tranche, and 10 refers to the bond maturity.

Results are reported in Table 2. Notice that the assumptions are those considered in [15, 16]. From the table we can observe how different tranching structures affect the zero-coupon yield rates.

Table 2 refers to January 31, 2019. In Figure 1 we consider the time evolution (2005-2019) of the different tranches considering 70-20-10 and 70-0-30 as tranching structure<sup>1</sup>. The figure shows that the yield rates of the different tranches were very similar in 2005. During the financial crisis, the junior tranche yield rate increased with a peak in 2012. After 2012, in a low interest rate environment, prices of sovereign bonds

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<sup>1</sup>For each year we consider the yield on January 31; if this day is a non-working day, then the first working day after January 31 is considered. GDP weights for the SBBS are computed considering a 10 years rolling period, e.g., weights for 2018 are computed considering the period 2007-2016.



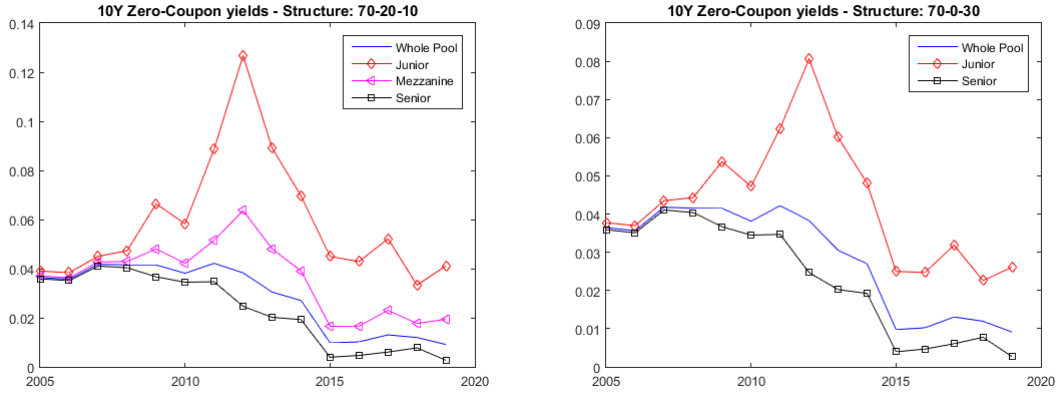


Figure 1: Term-structure of the 10Y zero coupon bonds yield rates.

Country	Weight	Country	Weight
Austria	2.97%	Ireland	1.99%
Belgium	4.56%	Italy	22.76%
Finland	1.22%	The Netherlands	4.68%
France	21.50%	Portugal	2.29%
Germany	23.43%	Spain	9.81%
Greece	3.57%		

Table 3: Average Public Debt over 2008-2017 as fraction of the EU area GDP.

started to include a default risk, the yield rates of the different tranches decreased but the dispersion among them increased. Notice that the rate of the whole pool (bond without tranching) is almost insensitive to the financial crisis and mimicks the time behavior of the senior tranche.

### 3 On the design of SBBS

In this section we analyze the robustness of the yield rates of SBBS tranches to design specifications in [15,16]. We will change the specifications and check the related impact on the yields.

Seniority	Senior	Mezzanine	Junior	Whole pool
70-20-10	0.36	2.59	4.53	1.12
80-10-10	0.57	3.07	4.51	1.12
70-0-30	0.36	-	3.19	1.12
80-0-20	0.57	-	3.78	1.12
90-0-10	0.81	-	4.50	1.12

Table 4: Yield rates of different tranches assuming debt weights and no recovery rate.

### 3.1 Debt weights

We compute SBBS weights deriving them from Public Debt of the countries (Refinitiv source, average over the period 2008-2017). The weights of the countries are reported in Table 3, and the total sum is 98.76%.

In Table 4 we report the yield rates of senior, mezzanine and junior tranches of SBBS in different settings considering Public Debt weights instead of GDP weights. Changing from GDP to debt weights on the composition of SBBS, the yield rates of the tranches change. As countries with a low credit quality also exhibit a significant public debt, debt weights lead to overweighting countries with low credit quality. Adopting debt weights leads to an increase of the yield rates of the tranches. The effect is limited for a SBBS without tranching (20 bps) and for senior tranches (around 10 bps), but it is significant for junior (40 bps) and, in particular, for mezzanine tranches (more than 60 bps in case of 70-20-10 tranching). Note that [21] considered a SBBS based on debt weights without tranching (whole pool) obtaining that the rating would be BBB-, if the senior tranche is 60% then the rating would be BBB+.

### 3.2 Recovery rate

In this section we analyze how yield rates of the tranches change assuming a positive recovery rate at default instead of a zero recovery rate as it is done in [15, 16]. Setting a recovery rate for sovereign bonds is a very difficult task. In what follows, we set the recovery rate at 0.38 for all states (0.4 for Greece), in line with the Refinitiv database. In Table 6 we report the yield rates of different tranches.

	AT	BE	FI	FR	DE	GR	IE	IT	NL	PO	ES
With recovery	0.94	0.91	0.95	0.93	1.00	0.50	0.89	0.64	0.97	0.78	0.82
No recovery	0.96	0.94	0.97	0.96	1.00	0.67	0.93	0.77	0.98	0.86	0.89

Table 5: Survival probabilities at 10Y computed from government bonds considering a positive recovery and a zero recovery rate.

As we are fitting our model with non zero recovery rates, instead of zero recovery rates, to the same zero coupon bond market prices, we will have different estimates for the implied survival probabilities with respect to those obtained in Section 2. In particular, we have that a non zero recovery rate implies a lower survival probability. Indeed, to justify the same price when increasing the recovery, or equivalently lowering the loss given default, one has to increase the default probability or, equivalently, to decrease the survival probability. We illustrate this with a little algebra for the reader convenience. Assuming a constant recovery rate and independence between the risk-free interest rate and the survival probabilities, the price of a zero coupon bond paying 1 in 10 years is given by

$$Z = Z^{rf} \times (PS + REC \times (1 - PS)),$$

where  $Z$  is the market price of the 10Y zero coupon bond,  $Z^{rf}$  is the price of the 10Y risk-free zero coupon bond,  $PS$  is the 10 years survival probability of the State issuing the bond, and  $REC$  is the recovery rate of the State. Therefore, we obtain:

$$PS = 1 - \frac{1}{REC - 1} \left( \frac{Z}{Z^{rf}} - 1 \right). \quad (1)$$

From this expression it turns out that a higher recovery rate corresponds to a smaller survival probability implied by the zero coupon bond price, as illustrated in Table 5.

Comparing Table 6 with Table 2 and Table 4, it is evident that a positive recovery rate has a significant effect on the yield rates of the tranches of SBBS. The yield rate of SBBS without tranching (whole pool) is not affected by a positive recovery rate. A positive recovery rate leads to a redistribution of default risk among tranches: the yield rate of junior tranches significantly increases (even 200 bps), the yield rate of senior tranches decreases (between 10 and 25 bps depending on the tranching structure), the yield rate of mezzanine tranches decreases in most of the cases.

Seniority	Senior	Mezzanine	Junior	Whole pool
	GDP weights, Non Zero Recovery Rate			
70-20-10	0.17	1.59	6.29	0.92
80-10-10	0.26	2.42	6.32	0.92
70-0-30	0.17	-	2.91	0.92
80-0-20	0.26	-	4.15	0.92
90-0-10	0.47	-	6.29	0.92
	Debt weights, Non Zero Recovery Rate			
70-20-10	0.21	2.49	6.73	1.12
80-10-10	0.34	3.85	6.73	1.12
70-0-30	0.21	-	3.70	1.12
80-0-20	0.33	-	5.18	1.12
90-0-10	0.67	-	6.72	1.12

Table 6: Yield rates of different tranches assuming GDP/debt weights and non zero recovery rate (38%).

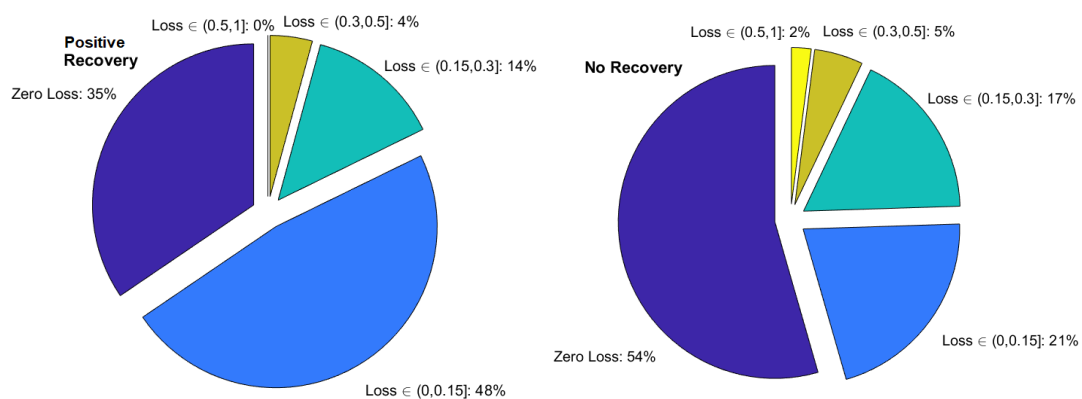


Figure 2: Loss distribution with and without recovery rate obtained by running 1.000.000 simulations. The loss is standardized, i.e., a loss of 1 corresponds to the underlying portfolio being worthless.

Let us investigate the mechanism in detail. A positive recovery rate induces two different effects: a) a non zero recovery rate makes single defaults more likely because

survival probabilities are lower, b) it limits the loss due to the default of any country. This makes the junior tranche riskier, while puts an upper bound on the loss making senior and mezzanine tranches safer. Figure 2 provides an illustration of the loss distribution with and without recovery rate running 1.000.000 simulations. We can observe that in the case of zero recovery rate, i.e., no recovery, there are more default-free simulations (therefore zero loss) than the case where a positive recovery is considered (54% vs 35%). However, considering the simulations where at least one default happens, we observe that the effect of the positive recovery rate is to make more likely small losses, with a 0 probability of observing losses larger than 0.5. We can assess that introducing a positive recovery rate we observe less simulations with a zero loss and more simulations with a small loss (below 0.30).

As a positive recovery rate is a natural assumption in this setting, the evidence provided in Table 6 suggests that the riskiness and the yield rate of junior tranches of SBBS have been underestimated in [15,16].

### 3.3 Correlation structure

In this section we deal with two different issues: the level of (uniform) correlation among defaults of States, and non uniform correlation of defaults among States. The analysis is performed in the baseline scenario: zero recovery rate and GDP weights.

Table 7 shows the yield rates of the tranches of SBBS changing the level of correlation in case of a uniform correlation matrix. We observe that the yield rate of the SBBS without tranching (whole pool) is not affected by the uniform correlation level because it is determined only by the zero coupon prices extracted from market data. The yield rate of senior tranches increases as the correlation increases, while the yield rate of junior tranches decreases; the effect on mezzanine tranches is ambiguous depending on the tranching structure. The assessment is that as the correlation among defaults increases, a risk shifting phenomenon is observed from junior to senior tranches because the scenarios with many (few) defaults is more (less) likely.

This effect can be depicted observing Figure 3. For a high correlation, there are more simulations with a loss below 15% and especially more zero loss simulations, hence making the junior tranche safer. On the other hand, we also observe more simulations with a loss higher than 30%, hence making riskier the senior tranche.

Seniority	Senior	Mezzanine	Junior	Whole pool
Uniform correlation 0.2				
70-20-10	0.18	1.84	5.46	0.92
80-10-10	0.26	2.95	5.45	0.92
70-0-30	0.18	-	2.89	0.92
80-0-20	0.26	-	4.12	0.92
90-0-10	0.53	-	5.47	0.92
Uniform correlation 0.4				
70-20-10	0.22	1.91	4.76	0.92
80-10-10	0.32	2.89	4.78	0.92
70-0-30	0.22	-	2.78	0.92
80-0-20	0.32	-	3.78	0.92
90-0-10	0.57	-	4.77	0.92
Uniform correlation 0.8				
70-20-10	0.33	1.94	3.50	0.92
80-10-10	0.45	2.64	3.51	0.92
70-0-30	0.33	-	2.43	0.92
80-0-20	0.45	-	3.05	0.92
90-0-10	0.67	-	3.50	0.92

Table 7: Yield rates of different tranches at various levels of uniform correlation. We assume GDP weights and zero recovery rates.

We now investigate how a different structure in the default correlation among states can alter the yield rates of different tranches. The assumption of a flat correlation structure is not realistic and it is not coherent with the financial crisis experience: [12] observed that during the financial crisis the correlation among bond returns of non core/core countries was high with some evidence of negative correlation between the two blocks.

In what follows, we still generate the defaults of the states through a Gaussian copula, but instead of adopting a uniform correlation matrix we consider a block structure. More

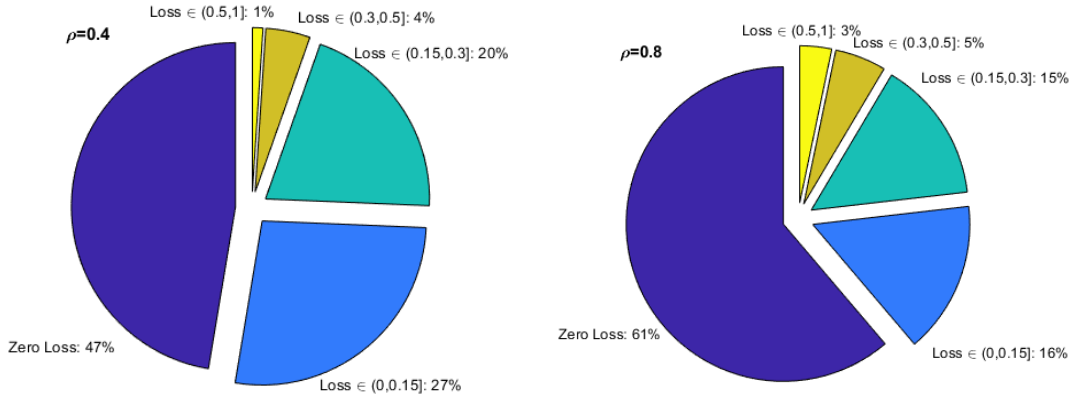


Figure 3: Loss distribution with different levels of correlation by running 1.000.000 simulations. The loss is standardized, i.e., a loss of 1 corresponds to the underlying portfolio being worthless. We refer to Figure 2 (right) for the case  $\rho = 0.6$ .

Seniority	Senior	Mezzanine	Junior	Whole pool
70-20-10	0.29	2.17	3.41	0.92
80-10-10	0.44	2.81	3.41	0.92
70-0-30	0.29	-	2.57	0.92
80-0-20	0.44	-	3.11	0.92
90-0-10	0.67	-	3.41	0.92

Table 8: Yield rates of different tranches assuming GDP weights and zero recovery rate. Correlation 1.00 inside block (Portugal, Italy, Ireland, Greece, and Spain), correlation 0.51 outside block, average correlation 0.6.

precisely, in Table 8 we assume a high uniform correlation (1.00) among non-core States (Portugal, Italy, Ireland, Greece, and Spain) and a low uniform correlation among the other States and among core and non-core States (0.51) in such a way that the average correlation among State defaults is 0.6 as in the benchmark case. Our hypothesis comes from the observation that countries with a high public debt are more likely to experience troubles (and defaults) together as they are exposed to the tightening of the markets and to flying to quality phenomena.

We observe that a block structure for the correlation among defaults impacts the

Seniority	Senior	Mezzanine	Junior	Whole pool
70-20-10	0.27	2.28	3.38	0.92
80-10-10	0.43	2.92	3.39	0.92
70-0-30	0.26	-	2.64	0.92
80-0-20	0.43	-	3.15	0.92
90-0-10	0.68	-	3.38	0.92

Table 9: Yield rates of different tranches assuming GDP weights and zero recovery rate. Two blocks: non-core and the others. Correlation inside block 1.00, correlation outside 0.27, average correlation 0.6.

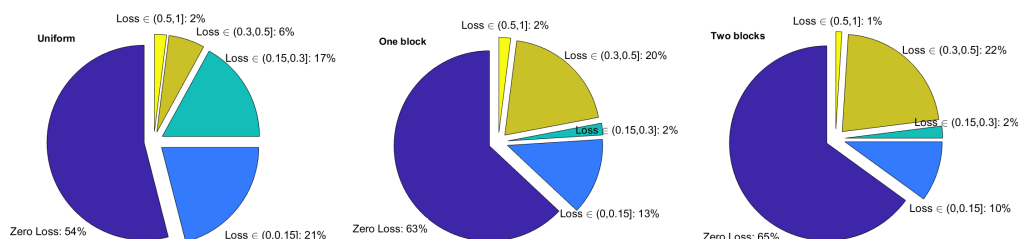


Figure 4: Loss distribution with different correlation structures running 1.000.000 simulations. The loss is standardized, i.e., a loss of 1 corresponds to underlying portfolio being worthless.

yield rates of the tranches. It depends on the tranching structure, but in general we observe a decrease of the yield of the junior tranche, an increase of the yield of senior and, in particular, of mezzanine tranches. The risk shifts from the junior tranches to the mezzanine ones and to a lesser extent to the senior ones as well.

We further expand our analysis to cover another example of block structure of the correlation matrix. In Table 9 we divide the countries in two blocks: non-core and other countries (Austria, Belgium, Finland, France, Germany, The Netherlands). Inside each block we assume a high correlation (1.00) and we assume a uniform correlation among countries of different blocks (0.27), see [9] for an analysis of correlation among sovereign bond markets in normal time and bad time. The results are similar to the single block one, shifting even more risk from junior to mezzanine and senior tranches.



The effect of correlation blocks is illustrated in the loss distribution plots in Figure 4. We observe that there are more default-free simulations and more simulations above 30% assuming one of the two block structures than in case of uniform correlation; considering losses between 15% and 30% we observe more simulations in case of uniform distribution than in case of a correlation structure with blocks (the difference is significant). We also notice that the loss distributions of the two different block structures are very similar. These results provide an explanation for risk re-distribution among tranches detected in the yield rates reported in Table 9.

In Figure 5 we plot the yield rates of senior, mezzanine and junior tranches over time in three settings: uniform 0.6 correlation, non-core block and non-core/core blocks. The yield rate of the senior tranche remains pretty stable with respect to the correlation structure, the yield rate of the mezzanine increases as we move from a uniform to a block correlation structure, in particular a significant increase is observed in case of a two block structure: a polarization among the defaults can cause a significant increase of the riskiness of mezzanine tranches. The yield rate of the junior tranche significantly decreases going from a uniform to a block correlation structure (an average decrease of approximately 0.83%), with a small effect moving from a one-block to a two-blocks correlation structure (an average decrease of 0.07%).

This result highlights that the yield rate (and therefore the riskiness) of the safe tranche of SBBS is robust to the strong correlation among defaults and is stable to stress, while the yield rate of the junior tranches become less risky and the yield rate of mezzanine tranches increases.

### **3.4 Recovery rate and correlation structure**

According to the previous analysis, a positive recovery rate and a higher correlation/block structure play opposite redistributive effects of risk among the tranches of SBBS. In this section we put them together (positive recovery rate and complex correlation structures) concentrating on the 70-20-10 architecture of SBBS. In Table 10 we report the results varying the correlation structure and assuming a non zero recovery rate. The results show that the positive recovery rate plays the dominant effect with a significant increase in the yield rate of junior bonds (always above 500 basis points) while the yield rate of the senior tranche, under different correlation structures, is always below 20 basis points.

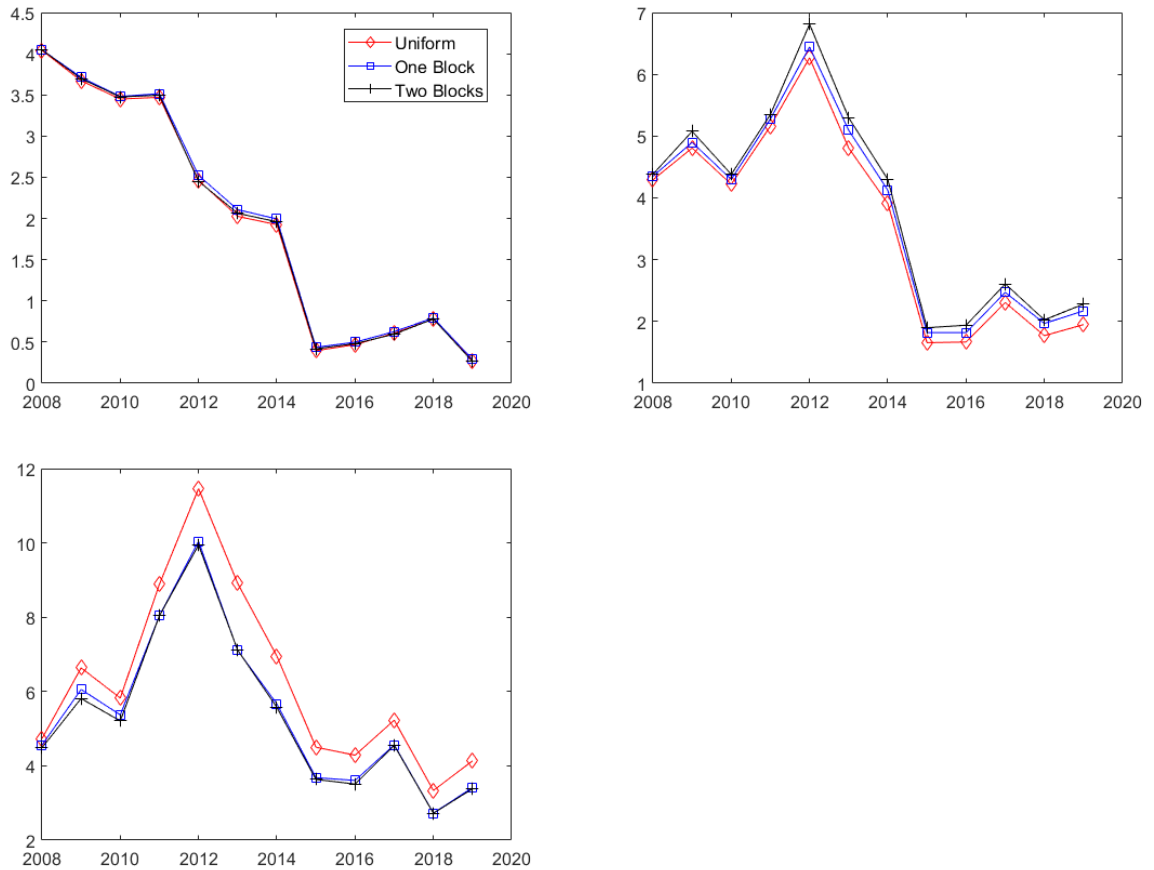


Figure 5: Yield rate (in percentage) of the senior (top, left), mezzanine (top, right) and junior (down, left) tranche across time for different correlation structures.

## 4 On the Gaussian Copula

All the analysis in [15,16] is based on the Gaussian Copula as a tool to model correlation among defaults. Essentially this model postulates that, given a pair of defaults, the joint default probability is obtained through a bivariate normal distribution having as arguments the default probabilities of the single names and a chosen correlation parameter for the dependence part. This approach can be generalized to numbers of entities larger than two. The point merits attention as the Gaussian copula model has been suggested to be one of the technicalities that are at the roots of the financial crisis, see [20,26]. Subsequent research has highlighted that while the model certainly has problems, the role of the model in the crisis has been somewhat exaggerated. This has been pointed out for example in [3]. Indeed [3], following an initial pre-crisis work in [2],

Seniority	Senior	Mezzanine	Junior	Whole pool	Correlation
70-20-10	0.14	1.27	7.96	0.92	Uniform $\bar{\rho} = 0.2$
70-20-10	0.15	1.45	7.05	0.92	Uniform $\bar{\rho} = 0.4$
70-20-10	0.17	1.59	6.29	0.92	Uniform $\bar{\rho} = 0.6$
70-20-10	0.20	1.71	5.53	0.92	Uniform $\bar{\rho} = 0.8$
70-20-10	0.14	1.71	6.35	0.92	One Block $\bar{\rho} = 0.2$
70-20-10	0.15	1.80	5.90	0.92	One Block $\bar{\rho} = 0.4$
70-20-10	0.17	1.86	5.51	0.92	One Block $\bar{\rho} = 0.6$
70-20-10	0.20	1.86	5.15	0.92	One Block $\bar{\rho} = 0.8$
70-20-10	0.14	1.75	6.30	0.92	Two Blocks $\bar{\rho} = 0.2$
70-20-10	0.15	1.79	5.99	0.92	Two Blocks $\bar{\rho} = 0.4$
70-20-10	0.18	1.81	5.61	0.92	Two Blocks $\bar{\rho} = 0.6$
70-20-10	0.21	1.80	5.25	0.92	Two Blocks $\bar{\rho} = 0.8$

Table 10: Yield rate of the 70-20-10 SBBS with non zero recovery subject to different correlation structure specifications.  $\bar{\rho}$  refers to the average correlation.

analyse the synthetic market of i-traxx tranches throughout the crisis and calibrate the market with a dynamic loss model. The multimodal and structured nature of the underlying loss distribution, highlighting default clustering, is discussed. This type of structure is absent if one uses a Gaussian copula with a flat correlation parameter across the matrix, and is hard to reproduce even when a Gaussian copula with richer parametrizations is used. In [3], referencing research published before the crisis in 2006, the authors show that the compound correlation paradigm, based on a flat correlation for each different tranche, may fail to fit a particular tranche price, whereas the base correlation paradigm, similarly based on a flat correlation for each different base tranche, may lead to negative expected losses. This points to the fact that while the Gaussian copula has limited appeal, the problems mostly come from the particular parametrization of this copula that has been used in the past, and with its inconsistent use across tranches for the synthetic i-traxx and CDX markets. In the end there was no theoretical argument in favour of using the Gaussian copula in the estimation of the probabilities of joint

defaults. The only advantage was in the widespread use of Gaussian distributions, and in the fact that for Gaussian distributions the total dependence across many names can be parametrized by a correlation matrix and be obtained by pairwise dependences given by single correlations of pairs. One further problem of the Gaussian copula, beside its arbitrariness, is that the related formula underestimates the probability of joint defaults under particularly severe conditions, unless the correlation is set to one. As a matter of fact, even admitting a certain degree of correlation between the Gaussian variables that represent the value of the issuers and which determine the default of the single position (one factor Gaussian model), the extreme values of these variables, which determine the default, are independent of each other (tail independence). As a consequence, it is impossible to use a Gaussian copula with correlations lower than one to reproduce the default clusters observed in the real world: when things go wrong, the defaults of bonds/loans all come together. Given the same probability of default for single entities, a collateralized debt obligation has a fatter loss tail than a bond issued by a company because if the defaults are interrelated, they are likely to occur together, yielding a higher probability of a significant loss.

This discussion suggests to look carefully at the modellization of the interaction of defaults. As a robustness check with respect to the dependence structure, we computed the yields of the different SBBS tranches in the case of a t-copula. In Table 11 we consider a 70-20-10 SBBS and we postulate that the defaults are correlated via a t-copula with uniform correlation equal to 0.6 and different degrees of freedom. As expected when the degrees of freedom grow, we get back the results of the Gaussian copula, since a t-distribution with many degrees of freedom approximates a Gaussian distribution. In Tables 12 and 13 we report respectively the yields of SBBS tranches when we assume the one and two blocks correlation structure considered in Section 3.3. We assume an average correlation of 0.6: comparing with the results reported in Tables 8-9, we observe the same convergence phenomenon increasing the degrees of freedom.

The analysis show that no significant effect is associated with changing the copula model: considering a t-copula, instead of a Gaussian copula, senior tranches become riskier with an increase of the yield rate of 3-5 basis points, and junior tranches become less risky with a decrease of 10-20 basis points in the yield rate.

Seniority	Senior	Mezzanine	Junior	Whole pool	Degrees of freedom
70-20-10	0.31	1.91	3.82	0.92	1
70-20-10	0.30	1.91	3.91	0.92	2
70-20-10	0.29	1.93	4.03	0.92	5
70-20-10	0.28	1.94	4.08	0.92	10
70-20-10	0.27	1.94	4.12	0.92	100
70-20-10	0.27	1.95	4.13	0.92	1000
70-20-10	0.27	1.95	4.13	0.92	10000

Table 11: Yield rate of the 70-20-10 SBBS with zero recovery, uniform correlation structure with average correlation of 0.6 and defaults correlated via a t-copula with different degrees of freedom.

Seniority	Senior	Mezzanine	Junior	Whole pool	Degrees of freedom
70-20-10	0.32	2.06	3.28	0.92	1
70-20-10	0.32	2.10	3.31	0.92	2
70-20-10	0.30	2.13	3.36	0.92	5
70-20-10	0.30	2.15	3.38	0.92	10
70-20-10	0.29	2.17	3.41	0.92	100
70-20-10	0.29	2.17	3.41	0.92	1000
70-20-10	0.29	2.17	3.41	0.92	10000

Table 12: Yield rate of the 70-20-10 SBBS with zero recovery, one block correlation structure with average correlation of 0.6 and defaults correlated via a t-copula with different degrees of freedom.

## 5 Safety without tranching

In [23] authors propose to build a safe asset without tranching, see also [18, 24]. They consider two main bond structures that do not foresee mutualization of debt/joint liabilities.

1. E-bonds, as proposed in [25]. The safe asset would be issued by a senior,

Seniority	Senior	Mezzanine	Junior	Whole pool	Degrees of freedom
70-20-10	0.31	2.16	3.28	0.92	1
70-20-10	0.29	2.20	3.31	0.92	2
70-20-10	0.28	2.24	3.35	0.92	5
70-20-10	0.27	2.26	3.36	0.92	10
70-20-10	0.27	2.29	3.40	0.92	100
70-20-10	0.27	2.30	3.40	0.92	1000
70-20-10	0.27	2.28	3.38	0.92	10000

Table 13: Yield rate of the 70-20-10 SBBS with zero recovery, two blocks correlation structure with average correlation of 0.6 and defaults correlated via a t-copula with different degrees of freedom.

publicly owned financial intermediary, backed by a diversified portfolio of sovereign bonds purchased at face value. In this setting, there is no tranching: seniority of the bonds is due to the nature of the intermediary. Note that private bond holders would be subordinated to E-bonds and therefore it is likely that the average cost of borrowing will increase. The weight of government bonds acquired is provided by the minimum between a fraction of GDP and a fraction of debt of the State (in the paper up to 50% of debt and 25% of GDP). The ratios are determined in order to maximize the volume of E-bonds under the assumption that the expected loss rate be equal to that of German bonds under the adverse scenario in [5].

2. National tranching followed by pooling as proposed in [24] and [28]. States issue senior and junior tranches, then senior tranches are pooled together according to GDP or Debt weights to build an asset backed security. The subordination level at national level is 70%: junior bonds represent 70% of the amount of bonds issued by States and senior bonds represent 30%.

In [24, Table 10], senior tranches of SBBS, national tranching and E-bonds are compared through the simulation approach of [5] (adverse scenario) computing the probability of default, Value at Risk (VaR), Expected Shortfall (ES). The order depends on the confidence level. It emerges that senior tranches of SBBS do a better job (they are less risky) in case of a small crisis but would perform in a bad way in case of a

systemic crisis: E-bonds and national tranching are riskier than senior bonds of SBBS (according to VaR) except for tail events occurring with less than 2% probability. The difference is due to the fact that a senior tranche of a SBBS would suffer a loss in case junior and mezzanine tranches have been entirely wiped out which entails that many countries should be in default. In case of E-bonds and national tranching, instead, the pool of senior tranches bears a loss as soon as one country exceeds the face value of junior bonds.

From a financial engineering point of view, national tranching and E-bonds look quite similar. In Table 14 we compare the yield rates of senior and junior tranches of SBBS and those of bonds associated with the National tranching architecture (the senior tranche obtained by pooling senior tranches of different States and junior tranches issued by the States). We recall that the reference yield rates of stand alone national bonds (without tranching) are: 2.72 (IT), 1.32 (ES), 0.59 (FR), 0.13 (DE). As a reference we can compare the yield rates of tranches of SBBS in case of 70-0-30 tranching and in case of National tranching with 30-0-70 tranching. Notice that the National tranching approach is successful in obtaining a safe asset but the drawback is the significant increase of the yield rate of junior bonds of some countries, i.e., +139 bps for Italy (from 2.72% to 4.11%), +48 bps for Spain (from 1.32% to 1.80%). Senior tranches of the two approaches provide similar yield rates.

This result agrees with what is observed in case of SBBS: pooling government bonds of different States to build a safe asset renders junior bonds extremely risky. The interesting and “surprising” result is that the yield rate of junior tranches of SBBS is significantly higher than the yield rate of junior bonds issued by Spain, France and Germany in case of pooling after tranching.

## 6 Loss statistics and Value at Risk

In Table 15 we show some quantile statistics for the loss over a ten year horizon of different tranches of SBBS. More precisely, we consider an architecture where the loss is absorbed by the junior (10%), then by the mezzanine (20%) and then by the senior tranche (70%).

Notice that, in the case of no recovery, junior and mezzanine tranches are wiped

Seniority	SBBS		National Tranching				
	Senior	Junior	Senior	Junior IT	Junior ES	Junior FR	Junior DE
30-0-70	0.13	1.27	0.13	4.11	1.88	0.79	0.13
40-0-60	0.13	1.47	0.19	4.74	2.12	0.87	0.13
50-0-50	0.13	1.77	0.43	4.74	2.12	0.87	0.13
60-0-40	0.14	2.22	0.59	4.74	2.12	0.87	0.13
70-0-30	0.17	2.91	0.70	4.74	2.12	0.87	0.13
80-0-20	0.26	4.15	0.79	4.74	2.12	0.87	0.13
90-0-10	0.47	6.29	0.86	4.74	2.12	0.87	0.13

Table 14: Yield rates of senior and junior tranches of SBBS, yield rates of senior tranches originated from National tranching and of junior tranches of single-state bonds tranced. GDP weights, positive recovery rate.

out in the worst 5% of scenarios over a 10 year horizon, while the senior tranche losses account between 1.5% and 8.3% depending on the correlation level and to a lesser extent on the structure of the correlation matrix. As the correlation increases, the risk measure goes up in particular for a low quantile (1% or 2%). As expected, the presence of a non zero recovery rate diminishes the senior tranche losses, in line with the results presented in Figure 2.

In what follows, we investigate the market risk of the different SBBS tranches. In particular, we report the VaR at different horizons and different percentiles of the prices of the different SBBS tranches. In Figure 6 we report the price time series. Prices are computed assuming a 100\$ maturity payment. As expected, the junior tranche is the one which suffered the most from the financial crisis. Notice that the mezzanine (junior) daily return is, on average, 1.93 (3.54) times higher than the senior tranche return. The same order holds true for the variance of the return of the mezzanine (junior) daily return, which is 1.74 (4.98) higher than that of senior tranche one.

In Figure 7 we show the 3 month VaR at 95, 98, 99% percentile computed using a 2 years rolling window. As expected, the Senior tranche is the safest (smallest VaR). Mezzanine and junior tranches are riskier than the senior tranche and their return



Quantile		5%		2%		1%	
Correlation Structure	$\mathbb{E}[L]$	$VaR(L)$	$ES(L)$	$VaR(L)$	$ES(L)$	$VaR(L)$	$ES(L)$
Uniform 0.4 no REC	7.5%	31.5%	41.2%	41.7%	50.9%	50.9%	56.7%
Uniform 0.6 no REC	7.5%	34.9%	48.0%	50.9%	59.1%	58.4%	63.5%
Uniform 0.8 no REC	7.5%	38.3%	55.0%	59.0%	65.1%	64.4%	68.2%
Uniform 0.6 REC	7.5%	27.4%	35.4%	36.7%	39.9%	39.6%	41.9%
One Block no REC	7.5%	35.7%	47.2%	49.4%	58.5%	57.1%	61.9%
One Block REC	7.5%	26.7%	34.8%	36.3%	38.9%	38.7%	40.9%
Two Blocks no REC	7.5%	33.6%	39.4%	39.6%	57.4%	59.0%	65.8%
Two Blocks REC	7.5%	23.3%	33.7%	36.7%	40.8%	40.0%	42.5%

Table 15: Statistics of the loss on the tranches underlying the SBBS over a 10 year horizon.

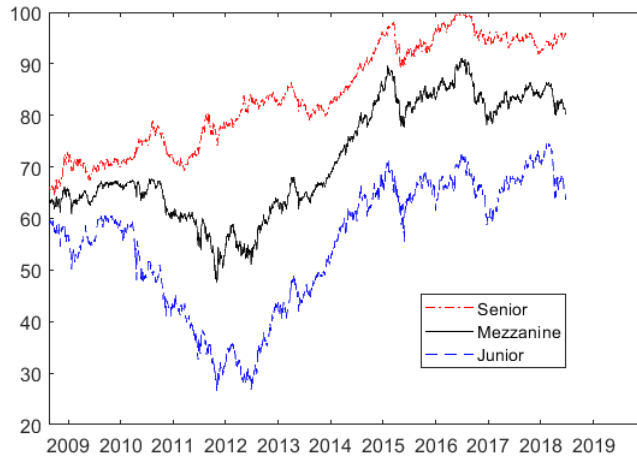


Figure 6: Prices of junior, mezzanine and senior tranches across time.

distribution has fatter tails as shown by the huge distance between the 95% and the 98% VaR.

If we consider the 98% VaR normalized by the tranche price, we obtain for the junior tranche an average value (variance) equal to 2.91 (35.79) times the average value (variance) of the senior tranche, while for the mezzanine the multiplicative factors are 1.83 (7.89). These results confirm that junior tranches are extremely risky with respect to the others.

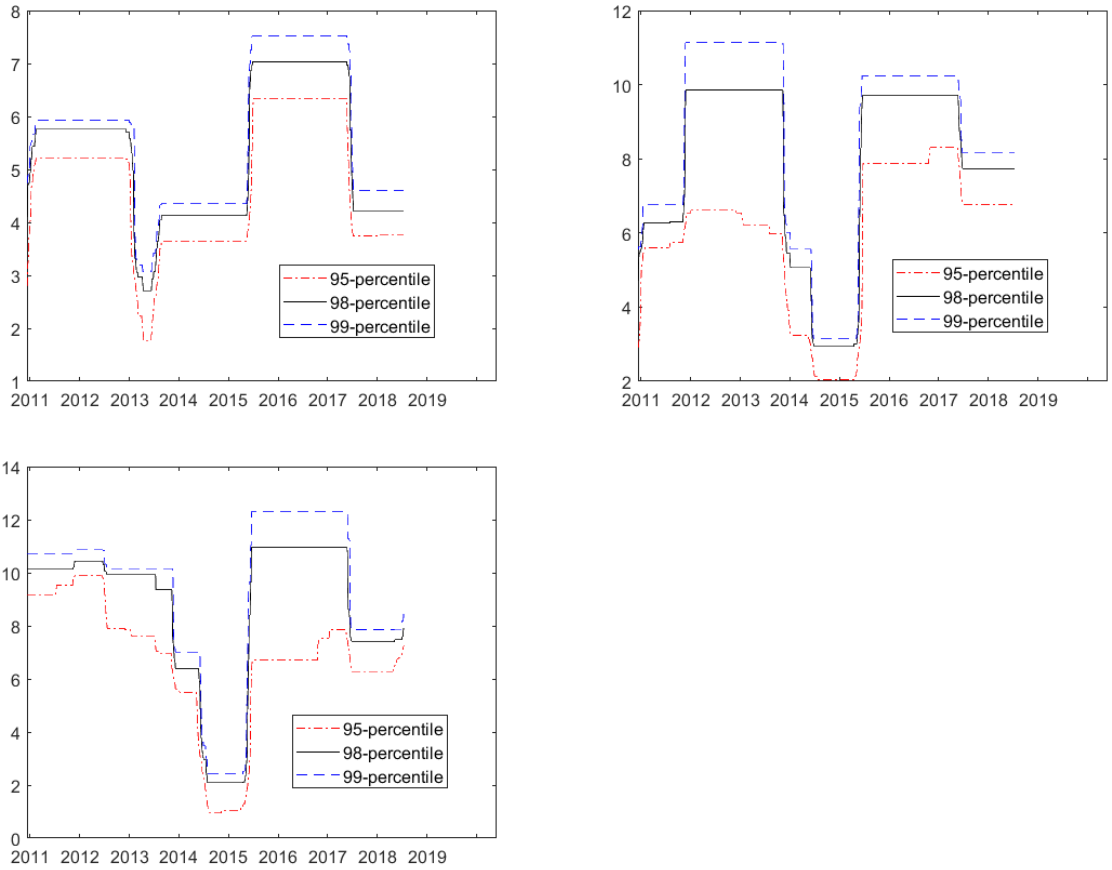


Figure 7: Value at Risk at 3 months for senior (top left), mezzanine (top right) and junior (down left) tranche.

## 7 Conclusions

The construction of a safe asset is a crucial issue in reforming the architecture of the Euro area. In this paper we have provided a robustness analysis of the SBBS proposal. We concentrate on yield rates of the different tranches and of their risk features. Our analysis showed that the correlation structure is not a big issue to obtain a safe asset and for the riskiness of junior bonds. Instead, the assumption of a zero recovery rate leads to a significant underestimation of the yield rate of junior bonds. Considering both a positive recovery rate and varying the correlation structure we observe that the two effects do not compensate each other: the yield rate of the safe tranche is low (below 20 basis points) but the yield rate of the junior tranche is quite high (above 500 basis points).

The high riskiness of junior bonds is a relevant issue as the yield rate of junior tranches is even higher than the one obtained for junior bonds of most of the states in case they issue senior and junior bonds. The point is critical because it highlights that the necessity to bail-out junior tranches of SBBS is not a remote event.

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